IEEE Conference on Control Applications
Toronto, Canada, August 2005.

Nonlinear H_∞ Controller Design for Flexible Joint Robots

H.D. Taghirad and M. Shaterian

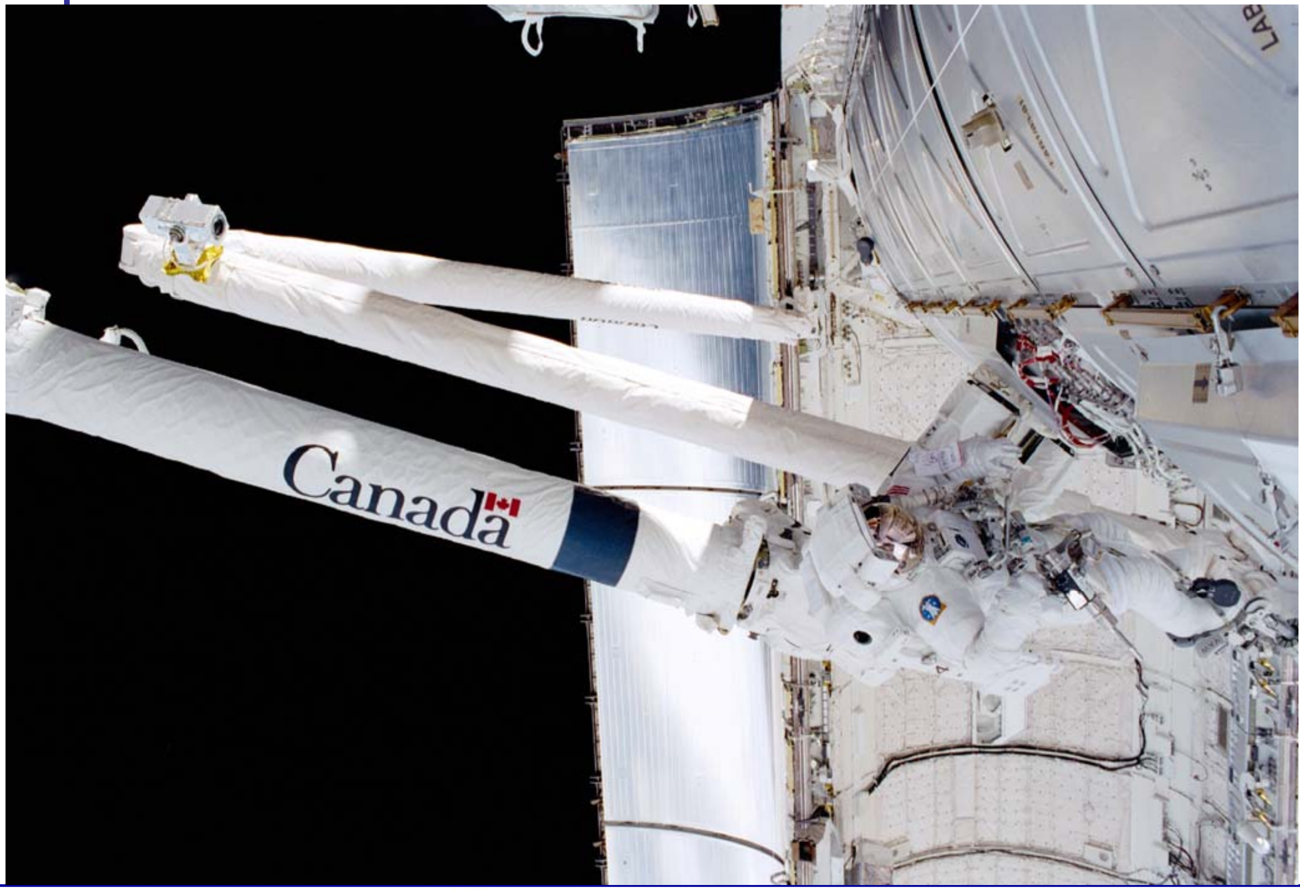
K.N. Toosi University of Technology,
Department of Electrical Engineering,
Advanced Robotics and Automated Systems,
Tehran, IRAN.

Outline

- Introduction
- Background
- FJR Modeling
- Nonlinear H_∞ Control
  - ✓ Problem Formulation
  - ✓ Uncertainty Incorporation
  - ✓ Tracking Objective
- Simulation Analysis
- Conclusions

Introduction

- Space Robotics



Background

- Modelling and Control
  - ✓Marino, 1984
  - ✓Spong, 1986
  - ✓Kokotovic, 1987
  - ✓Ghorbel, 1991
  - ✓Khorasani, 1993
  - ✓Elmaraghi 1999
- Linear and Robust
  - ✓Reshmin, 2000
  - ✓Schaeffer, 2000
  - ✓Thummel, 2001
  - ✓Bakhshi, 2003
  - ✓Ozgoli 2004

FJR Modeling

- Rigid Model:

$$\begin{cases} \tau = M(q)\ddot{q} + N(q,\dot{q}) \\ N(q,\dot{q}) = V_m(q,\dot{q})\dot{q} + G(q) + F_d\dot{q} + F_s(\dot{q}) + T_d \end{cases}$$

- Dynamics Features
  - ✓ Mass matrix pos. def.
  - ✓ $\dot{M}(q) - 2V_m(q,\dot{q})$ is skew symmetric
  - ✓ Friction terms are dissipative

$$y^T(F_d y + F_s(y)) \geq y^T F_d y \geq k_f \|y\|^2$$

  - ✓ Revolute joint property

$$L(q + 2k\pi, \dot{q}) = L(q,\dot{q})$$

  - ✓ boundedness

$$m_1 I \leq M(q) \leq m_2 I$$

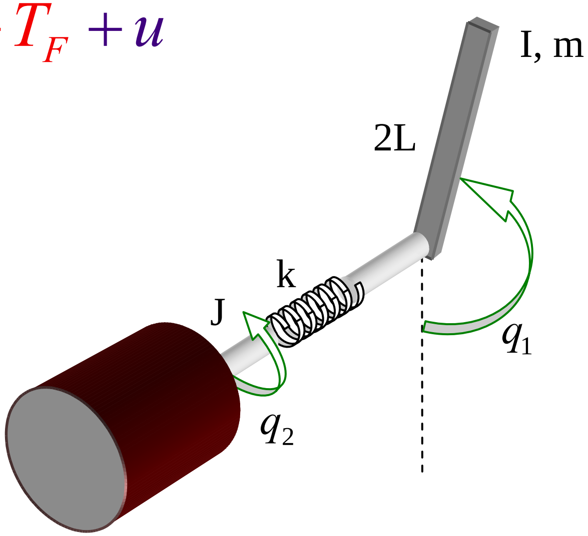
$$V_m(q,\dot{q}) \leq \xi_c \|\dot{q}\|$$

$$\vdots$$

FJR Modeling

- Flexible Model:

$$\begin{cases} M(q_1)\ddot{q}_1 + N(q_1,\dot{q}_1) = K(q_2 - q_1) \\ J\ddot{q}_2 = K(q_2 - q_1) - D\dot{q}_2 + T_F + u \end{cases}$$



- Where

q_1 = Link Position

q_2 = Actuator Position

✓ $N(q_1,\dot{q}_1) = V_m(q_1,\dot{q}_1)\dot{q}_1 + G(q_1) + F_d\dot{q}_1 + F_s(\dot{q}_1) + T_d$

Nonlinear H∞ Controller Synthesis

- Problem Formulation
  - ✓ System model: $\dot{x} = X(x, w, u), \quad z = Z(x, w, u)$
  - ✓ State Feedback Controller $u = a_2(x)$

$$\dot{x} = X(x, w, a_2(x)) \quad z = Z(x, w, a_2(x))$$

  - ✓ Objectives:
    - Robust Stability
    - Disturbance Rejection

Optimize: $\dfrac{\|z\|_2}{\|w\|_2} = \dfrac{\sqrt{\int_0^\infty z^T(t) z(t) dt}}{\sqrt{\int_0^\infty w^T(t) w(t) dt}} < \gamma$

  - ✓ Approximate Solution of HJI

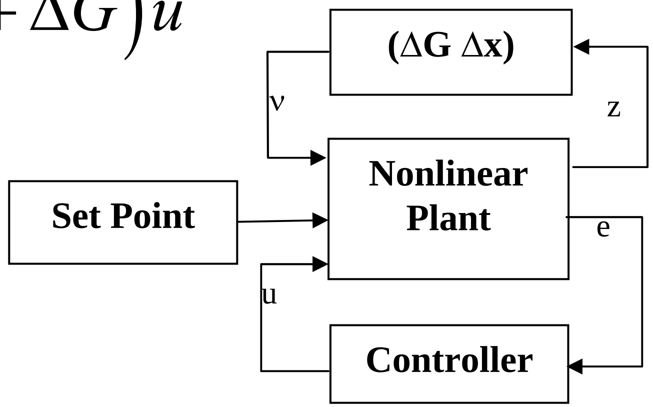
Nonlinear H$_\infty$ Controller Synthesis

- Incorporation of Uncertainties
  - ✓ **Yazdanpanah 1997**

$$\dot{x} = \bar{f}(x) + \Delta f(x) + \left(\bar{G} + \Delta G\right)u$$

$$\dot{x} = \bar{f}(x) + \bar{G}u + v$$

$$v = \Delta f + \Delta G \times u$$

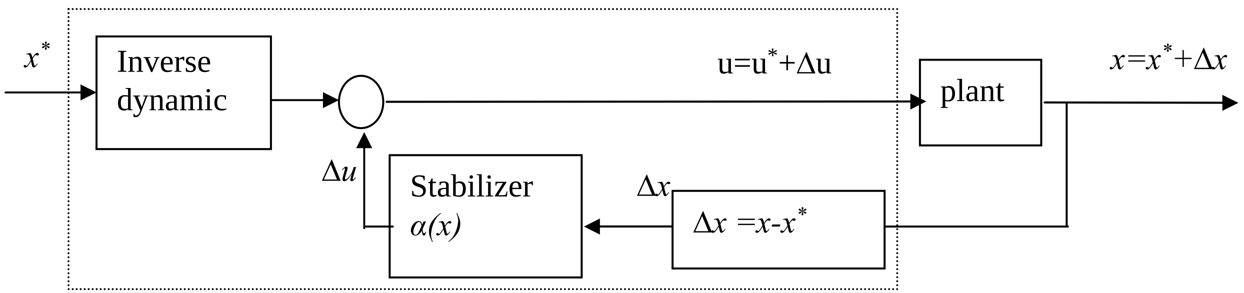


$$v = \begin{pmatrix}\Delta G & \Delta(x)\end{pmatrix}\begin{pmatrix}u \\ x\end{pmatrix} \quad \Delta(x) := \int_0^1 \frac{\partial \Delta f(\lambda x)}{\partial x} d\lambda$$

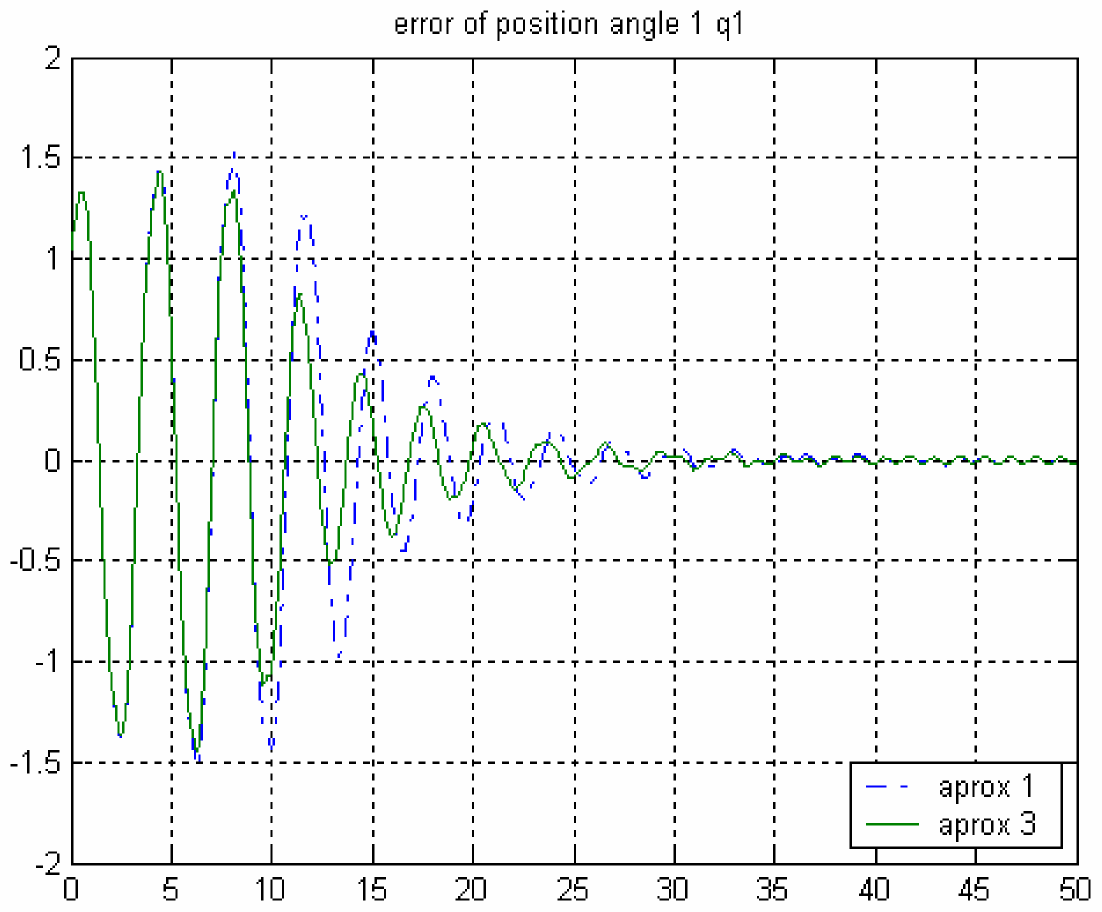
Nonlinear H∞ Controller Synthesis

- Tracking Objective
  - ✓ Linearize about the desired trajectory x^*

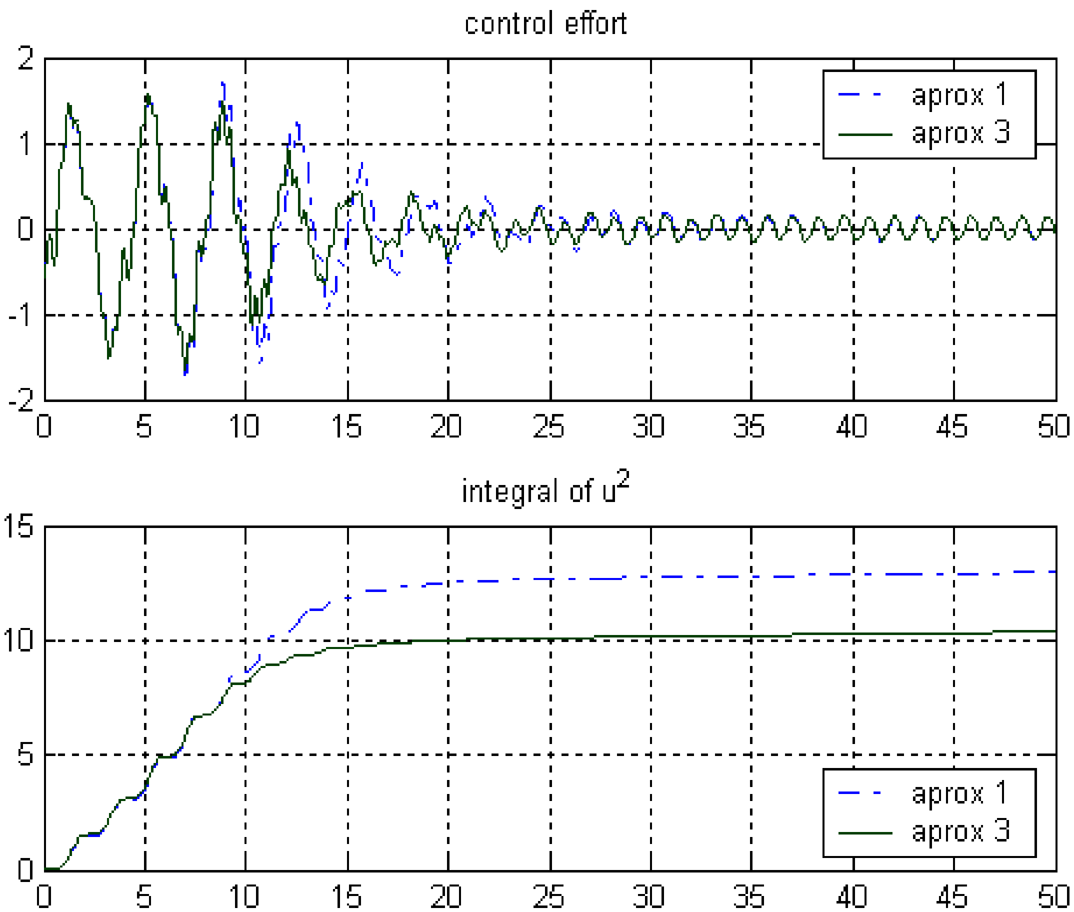
$$\dot{x}^* = f\left(x^*, u^*\right) \qquad \Delta\dot{x} + \dot{x}^* = f\left(x^* + \Delta x, u^* + \Delta u\right)$$



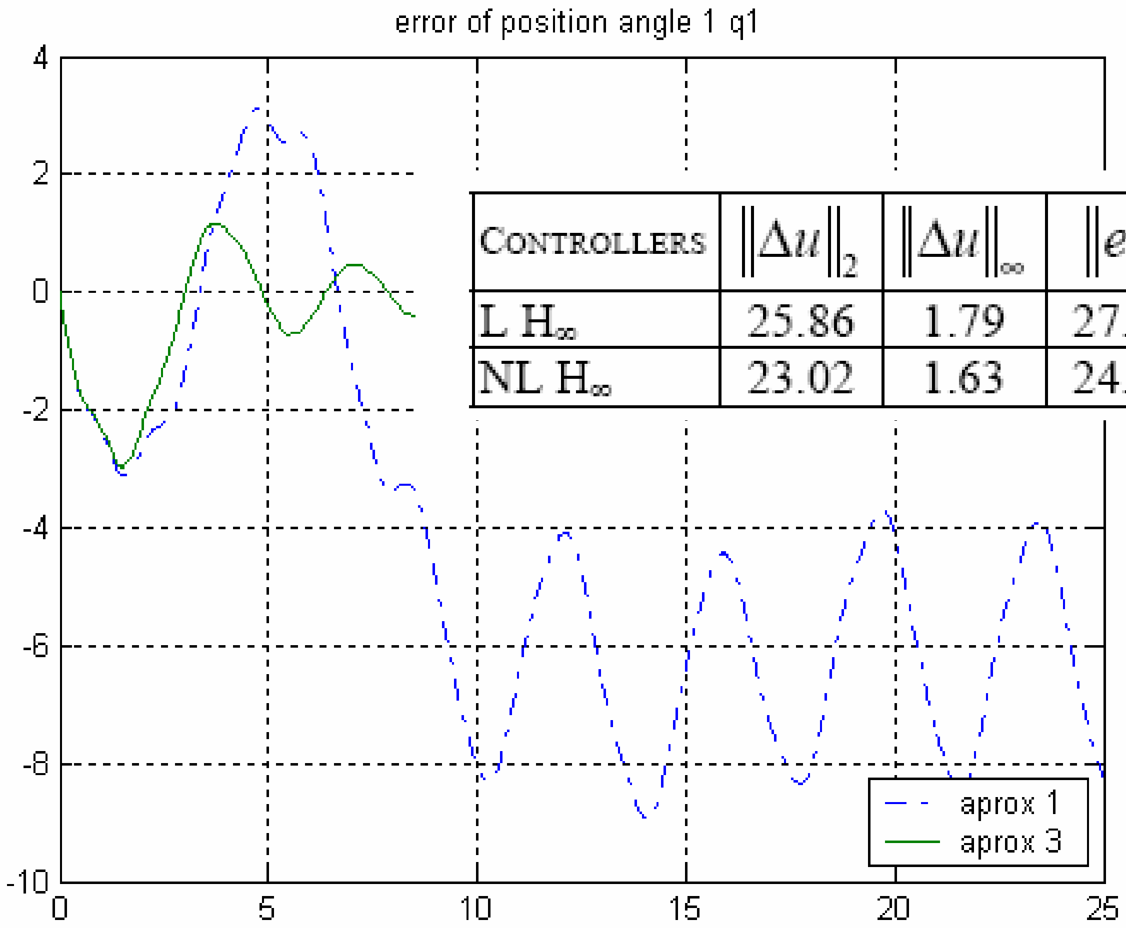
Simulation Analysis



Simulation Analysis



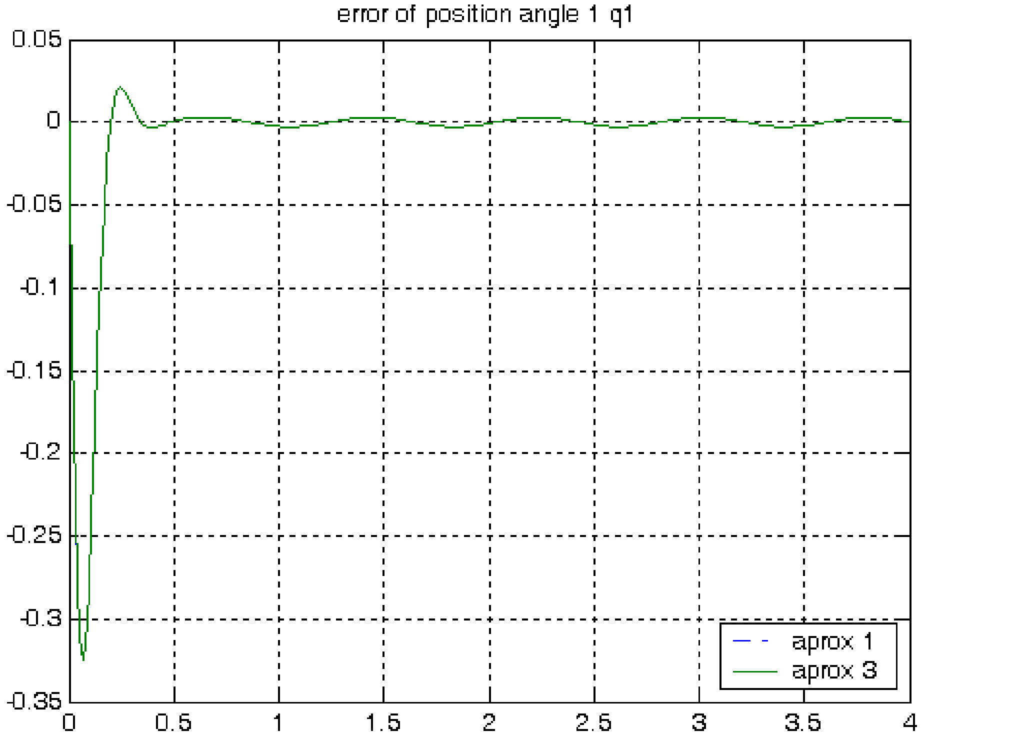
Simulation Analysis



CONTROLLERS	$\|\Delta u\|_2$	$\|\Delta u\|_\infty$	$\|e\|_2$	$\|e\|_\infty$
L H_∞	25.86	1.79	27.74	1.52
NL H_∞	23.02	1.63	24.03	1.44

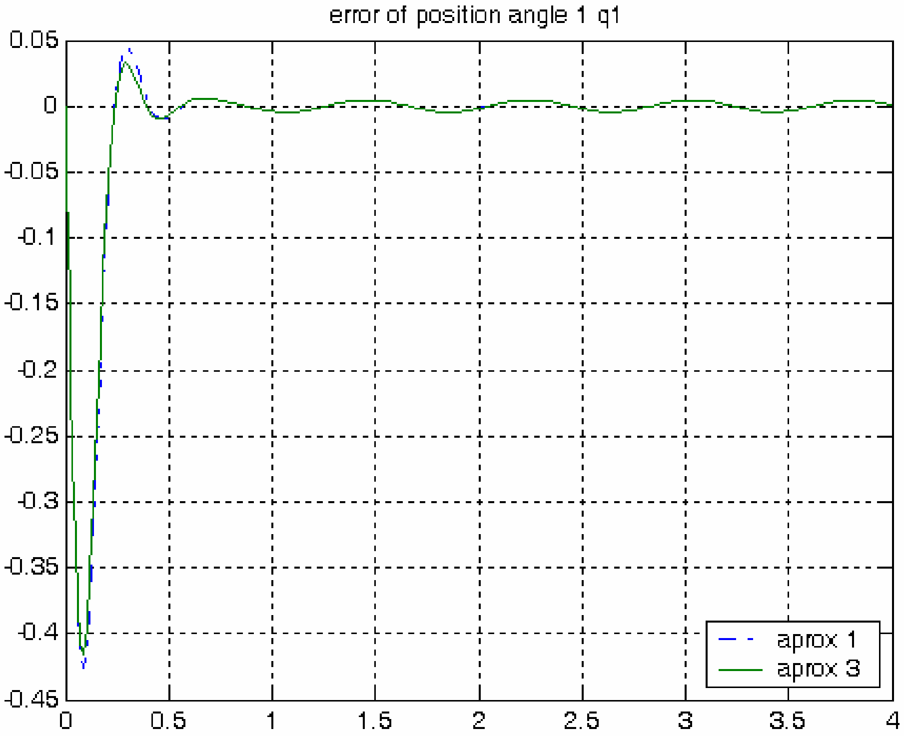
Simulation Analysis

- High Performance Controller
  - ✓ Penalty variable $z = [10000x_1, 100x_2, 100x_3, 100x_4, u]$



Simulation Analysis

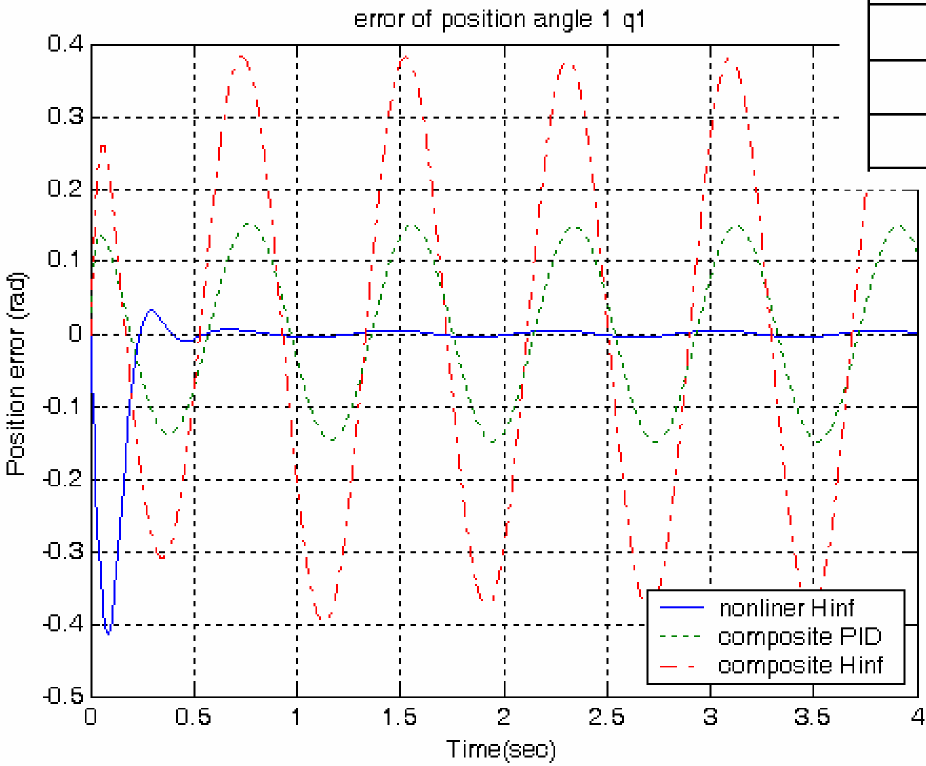
- High Performance Controller
  - ✓ Penaltv variable $z = \left[5000x_1 + 12000x_1^2 + 15000x_1^3, 100x_2, 100x_3, 100x_4, u\right]$

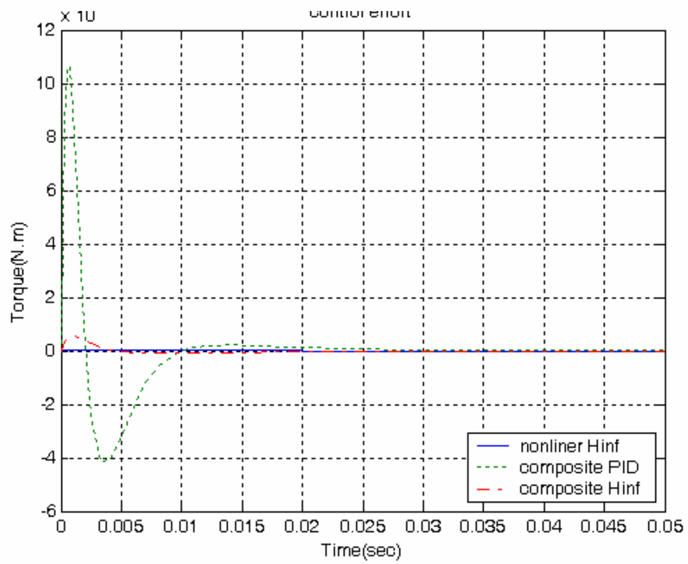


Comparison Analysis

- Composite PID and Composite H$_\infty$ and Nonlinear H$_\infty$

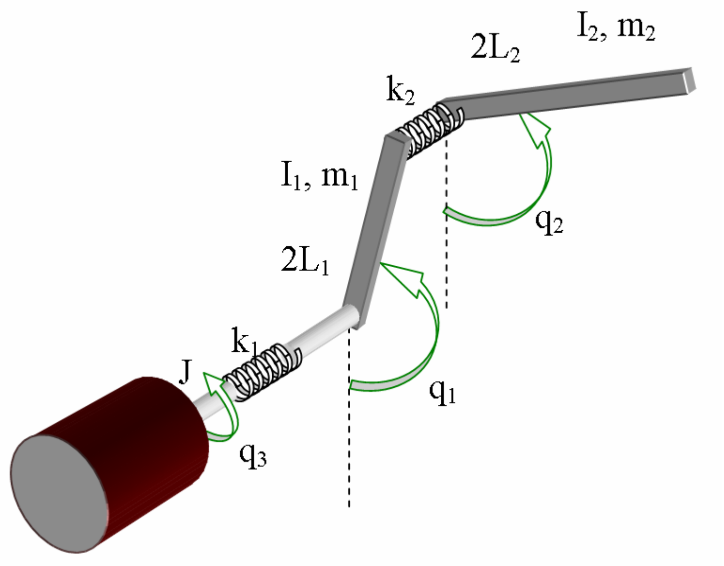
Control method	$\|e\|_2$	$\|e\|_\infty$	$\|u\|_\infty$
Composite PID	0.21	0.152	10.7×10^5
Composite H$_\infty$	0.52	0.39	0.54×10^5
Nonlinear H$_\infty$	0.132	0.41	0.04×10^5

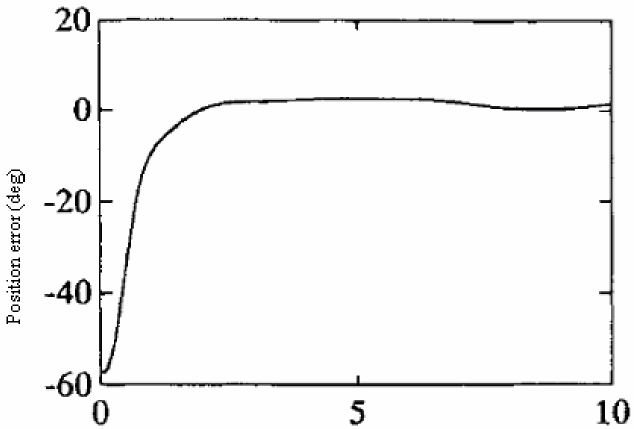


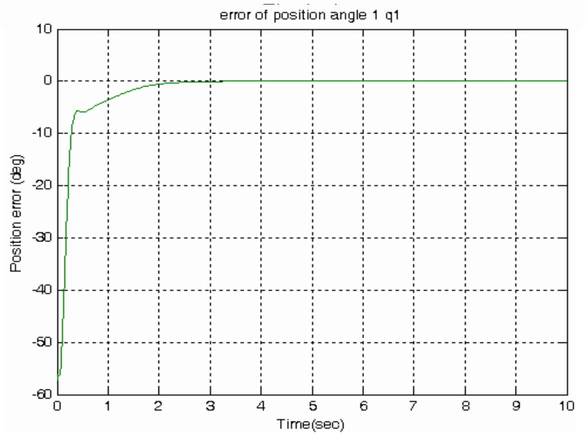


Comparison Analysis

- Two Link FJR







Conclusions

- Implementation of Nonlinear H_∞ Controller for FJR
  - ✓ Uncertainty Incorporation
  - ✓ Tracking Performance
- Simulation Analysis
  - ✓ Stability properties
  - ✓ Performance compared to other controllers
- Extendable to Multi Link FJR

Visit our group site:

http://saba.kntu.ac.ir/eetd/taghirad

Thank You