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# QFT Controller Synthesis For A Nonlinear Flexible Joint Robot

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# Outline

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- Introduction
- Nonlinear QFT
- QFT Controller design for FJR
- Composite QFT Design
- Simulation Results
- Conclusions



# Introduction

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- FJR Applications
  - ✓ Space Robotics
  - ✓ Micro Robots
  - ✓ Dextereous Robotic Hand
  - ✓ Industrial Robots with Harmonic Drive
  - ✓ . . . .



# Background

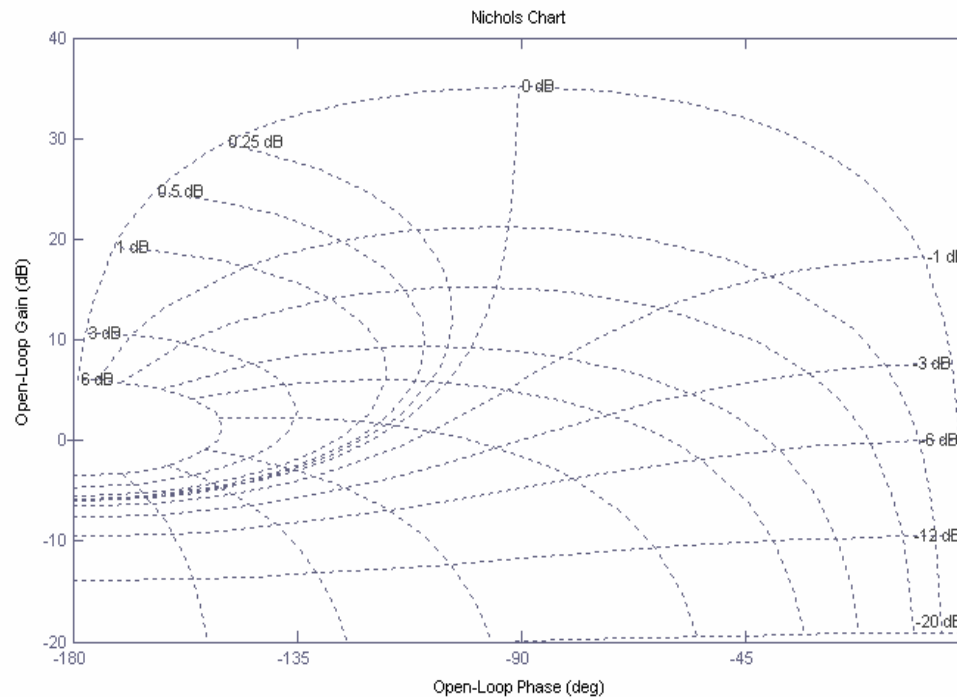
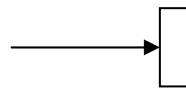
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- Marino, 1984
- Spong, 1986
- Kokotovic, 1987
- Ghorbel, 1991
- Khorasani, 1993
- Elmaraghi 1999
- Reshmin, 2000
- Schaffer, 2000
- Thummel, 2001
- Bakhshi, 2003
- Shaterian 2005
- Ozgoli 2005



# QFT Overview

- Robust Control
- Two Degree



- Frequency Domain (Nichols Chart)



# QFT Overview

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- Design Steps
  - ✓ Templates
  - ✓ Bounds
    - Stability
    - Tracking
    - Disturbance
  - ✓ FD Loop Shaping
  - ✓ Prefilter Design
  - ✓ FD Performance Check



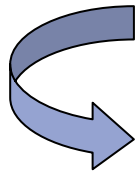
# Nonlinear QFT

- Nonlinear Equivalent Disturbance Attenuation
- Equivalent LTI Family

$A_R = \{A_r \mid r \in R\} \longrightarrow$  **set of acceptable output**

$P = \{p_\theta : U \rightarrow Y, \theta \in \Theta\} \longrightarrow$  **NLTV system**

$U_{A,r} := \{u \in U \mid u = p_\theta^I(y), y \in A_r\} \longrightarrow$  **set of acceptable input**



**Equivalent  
LTI Family**

$$P_\theta(A_R) := \{p_{r,\theta}^y \mid y \in A_r, r \in R\}$$

$$P(A_R) := \{P_\theta(A_R) \mid \theta \in \Theta\} = \{p_{r,\theta}^y \mid y \in A_r, r \in R, \theta \in \Theta\}$$



# QFT Controller Synthesis: Rigid Robot

Suppose rigid manipulator of:  
Find  $F(s)$  &  $G(s)$  such that:

$$\ddot{q} = -4.9 \text{Sin}q + \frac{1}{2}u$$

a) Stability margin: If  $L(s) = P(s) * G(s)$  then

$$\left| \frac{L(j\omega)}{1 + L(j\omega)} \right| \leq M = 1.2 \quad \forall \omega \in [0, \infty]$$

b) Tracking Condition:  $|T_\ell(j\omega)| < \left| F(j\omega) \frac{L(j\omega)}{1 + L(j\omega)} \right| \leq |T_u(j\omega)| \quad \forall \omega \in [0, \infty]$

where

$$\begin{cases} T_u(s) = \frac{c_1 c_2 c_3 c_4 (s + a) / a}{s(s + c_1)(s + c_2)(s + c_3)(s + c_4)} & a = 5 \\ T_\ell(s) = \frac{c_1 c_2 c_3 c_4 (s + a) / a}{s(s + c_1)(s + c_2)(s + c_3)(s + c_4)} & a = 200 \end{cases}$$



# QFT Controller Synthesis: Rigid Robot

set of acceptable output:

$$Y(s) = \frac{c_1 c_2 c_3 c_4 (s + a) / a}{s(s + c_1)(s + c_2)(s + c_3)(s + c_4)}$$

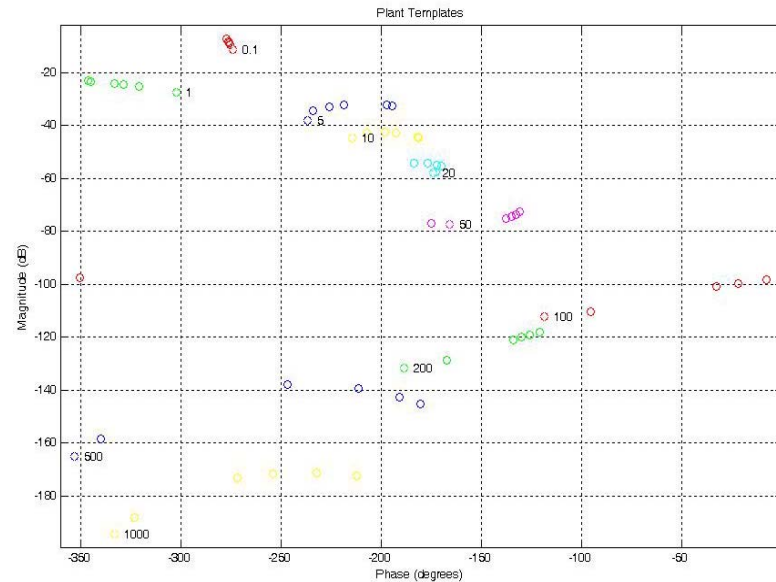
where  $c_1=20$  ,  $c_2=21$  ,  $c_3=22$ ,  $c_4=25$  and  $a=[5,10,15,20,100,200]$

$$u = 2\ddot{q} + 9.8 \sin q$$

$$Ltifam(j\omega_i) = \frac{Y(j\omega_i)}{U(j\omega_i)}$$

Inverse Equation

0.1,r
1,g
5,b
10,y
20,c
50,m
100,l
200,g
500,b
1000,y
Off

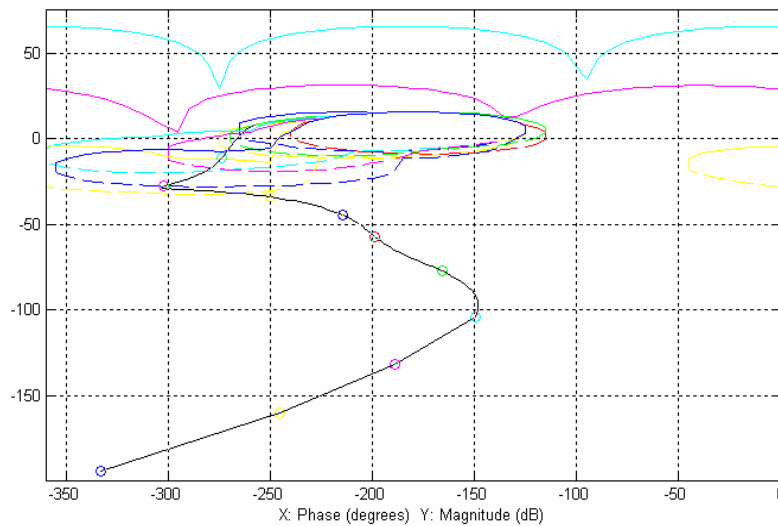


- Templates

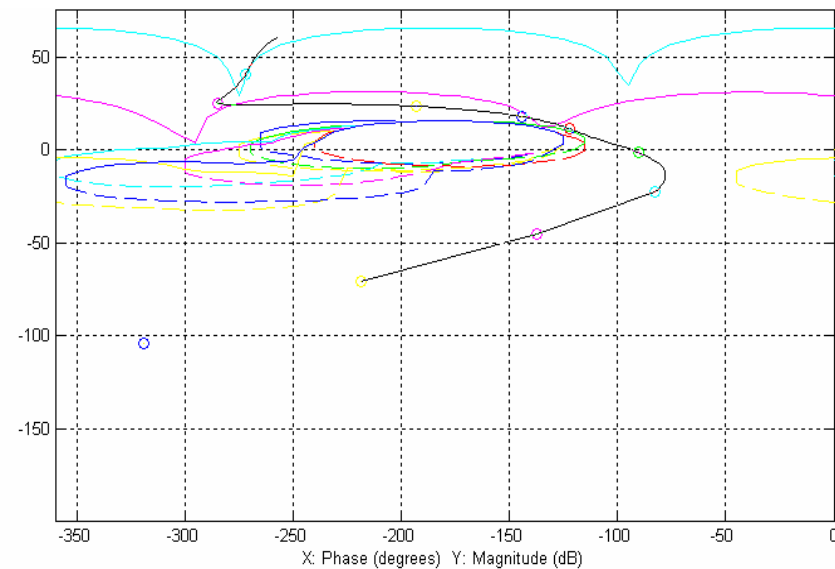


# QFT Controller Synthesis: Rigid Robot

- Loop Shaping



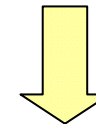
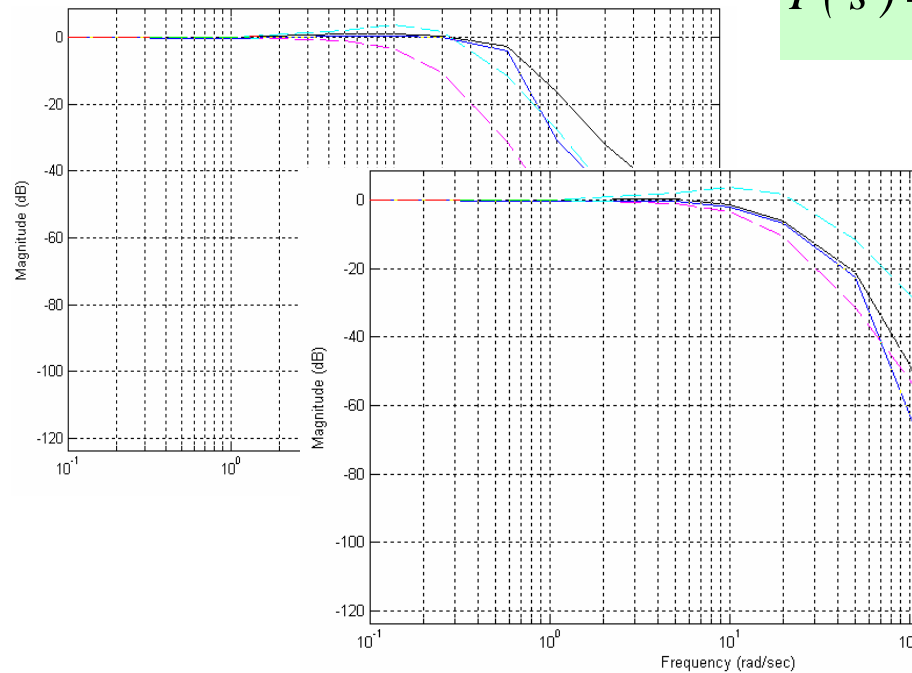
$$G(s) = \frac{3.296 \times 10^4 s + 1.033 \times 10^5}{s + 257.2}$$



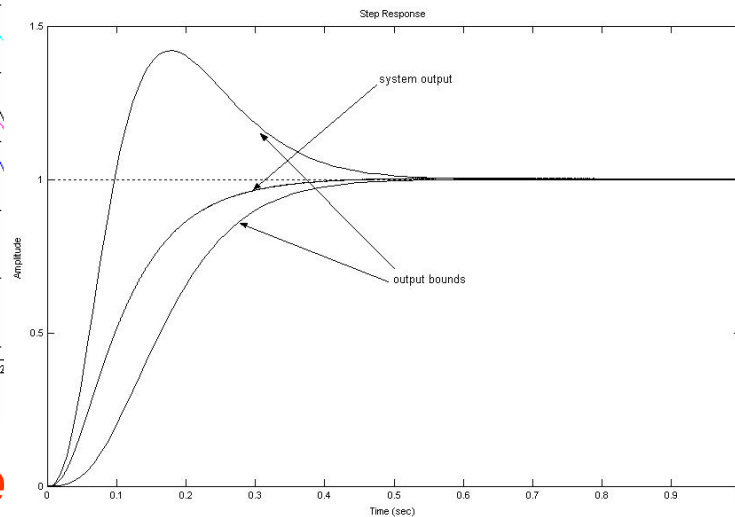
# QFT Controller Synthesis: Rigid Robot

- Prefilter Design

$$F(s) = \frac{3.172 * 10^4}{s^3 + 112 s^2 + 3720 s + 3.146 * 10^4}$$



&  $G(s)$

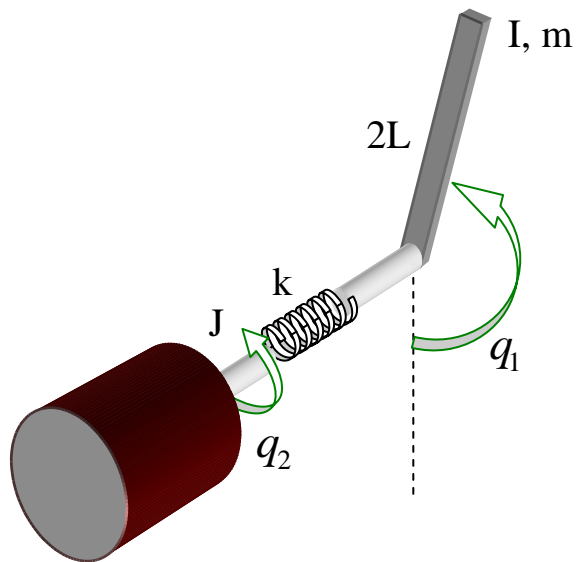


**Step Response**



# QFT Controller Synthesis: Flexible Joint Robot (FJR)

- FJR Model



$$I=J=m=L=1 \quad g=9.8, \quad k=100$$

## Dynamics

$$\begin{cases} I\ddot{q}_1 = k(q_2 - q_1) - mgL \sin q_1 \\ J\ddot{q}_2 = u - k(q_2 - q_1) \end{cases}$$

$$q=q_1 \quad z=k(q_1-q_2), \quad \varepsilon=1/k$$

$$\begin{cases} \ddot{q} = \left(-\frac{1}{I}\right)z - \frac{mgL \sin q}{I} \\ \varepsilon\ddot{z} = \left(-\frac{1}{I} - \frac{1}{J}\right)z - \frac{mgL \sin q}{I} - \left(\frac{1}{J}\right)u \end{cases}$$

## Singularly Perturbed Model

$$\begin{cases} \ddot{q} = -9.8 \sin q - z \\ (0.01)\ddot{z} = -2z - 9.8 \sin q - u \end{cases}$$

# QFT Controller Synthesis: FJR

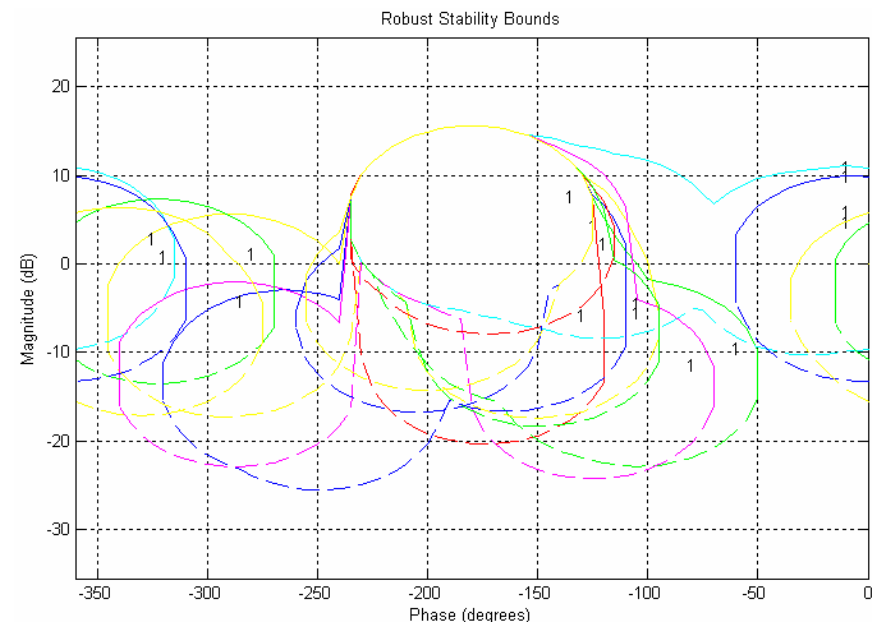
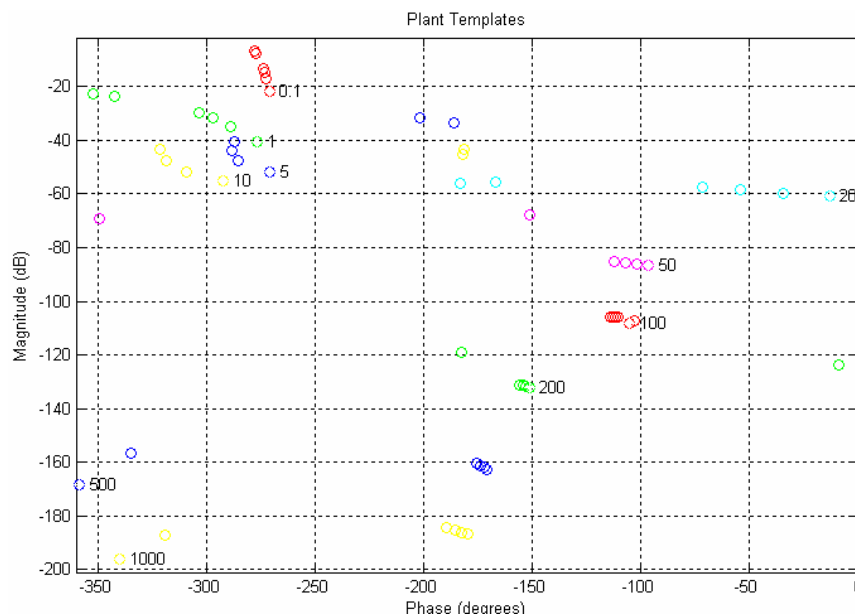
- Templates

*Inverse model of FJR:*

$$u = (\ddot{q}^{(4)} + (9.8 \cos q + 2k)\ddot{q} - 9.8 (\sin q)(\dot{q})^2 + 9.8k \sin q) / k$$



***What causes large templates?***

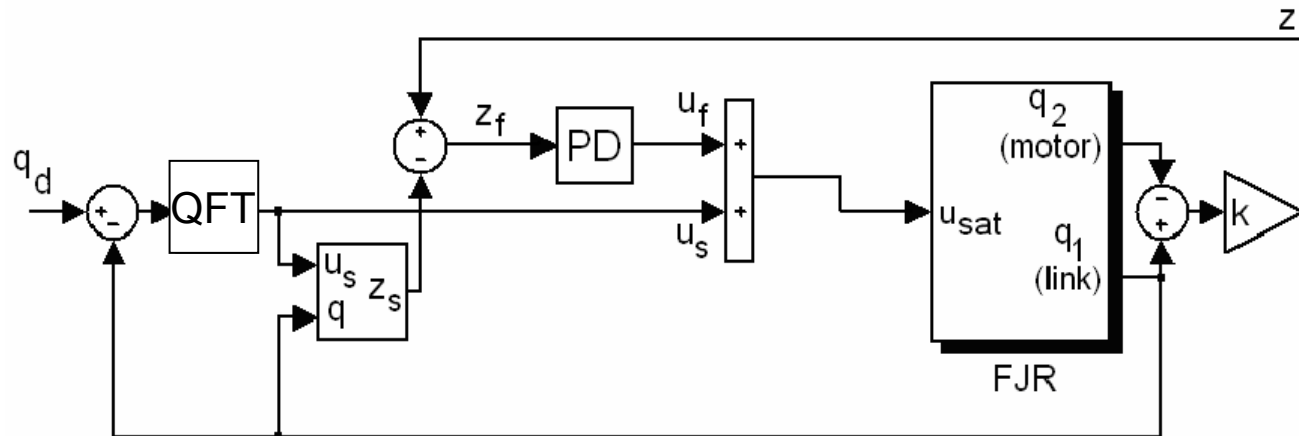


# Composite QFT Design on FJR

- Composite Controller

$$u = u_s(q, \dot{q}, \varepsilon) + u_f(\eta, \dot{\eta})$$

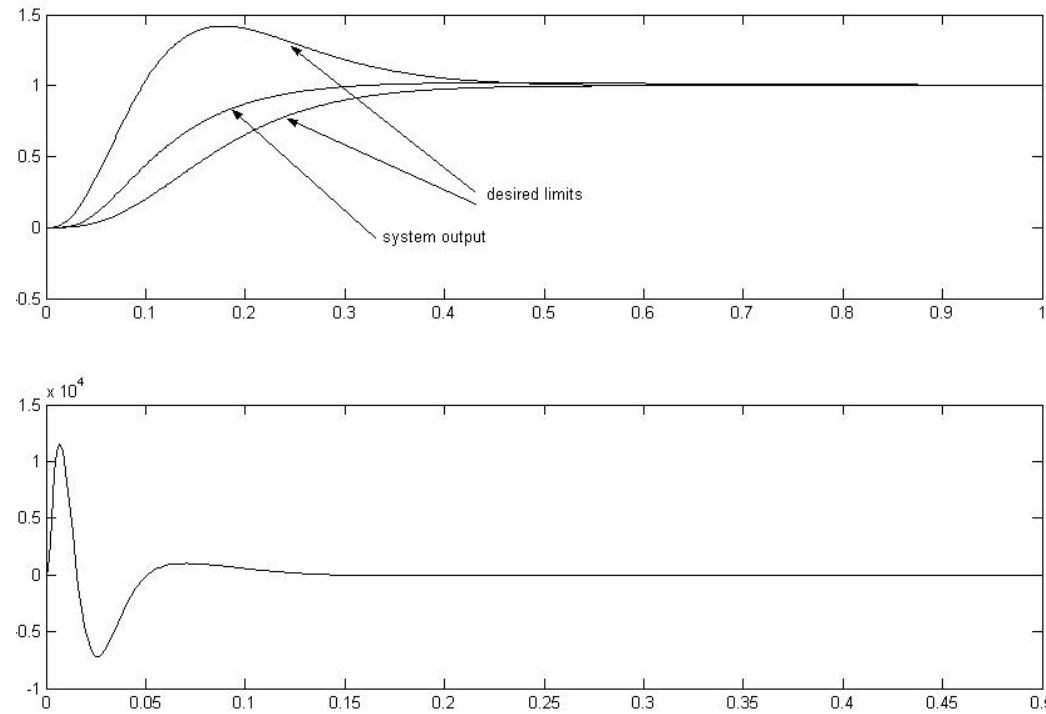
- Fast Controller: PD + Corrective term (IM)
- Slow Controller: QFT



# Composite QFT Design on FJR

- Rigid Controller

$F(s) & G(s)$

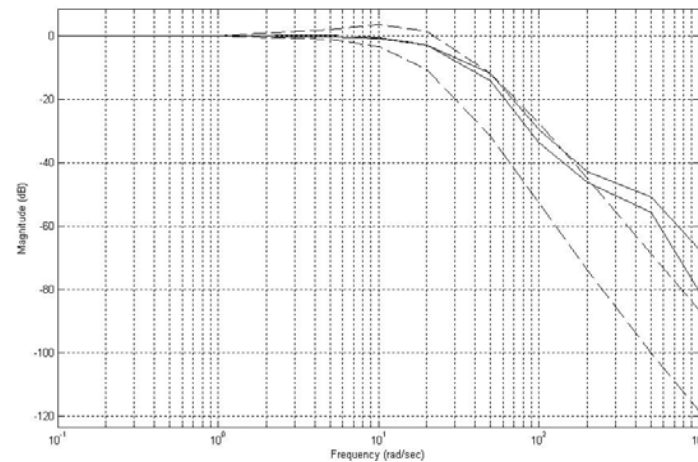
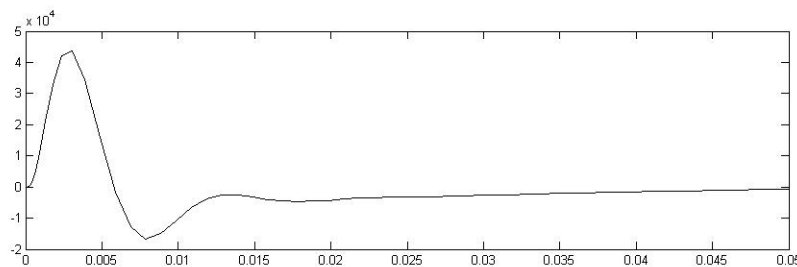
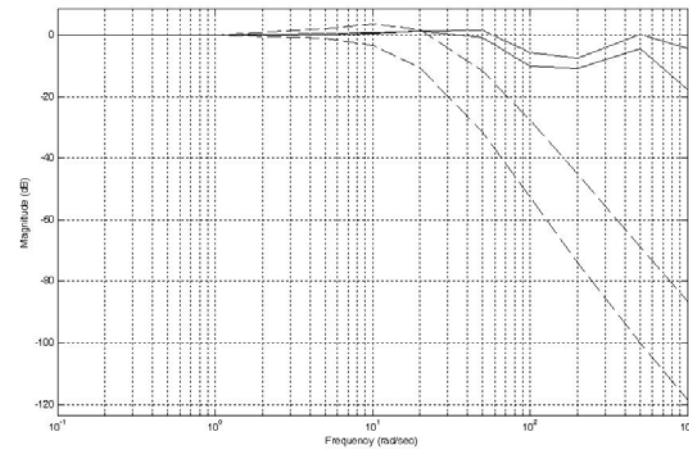
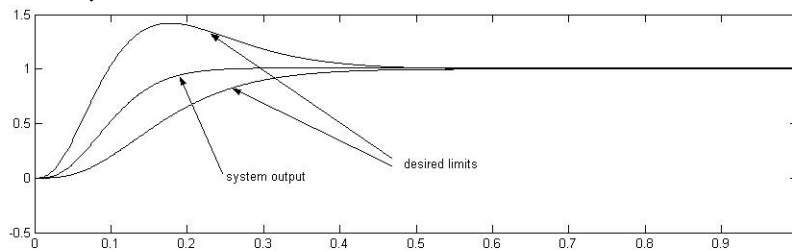


- Composite Controller



# Composite QFT Design on FJR

$$G(s) = \frac{1644765.504(s + 69.3)(s + 36.83)}{(s + 1861)(s + 577.2)}$$



$$F_1(s) = \frac{24220.0786}{(s + 34.59)(s + 32.1)(s + 21.72)}$$

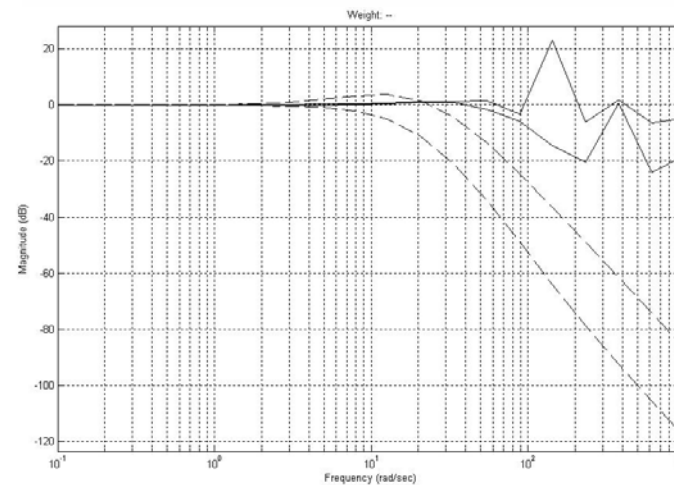
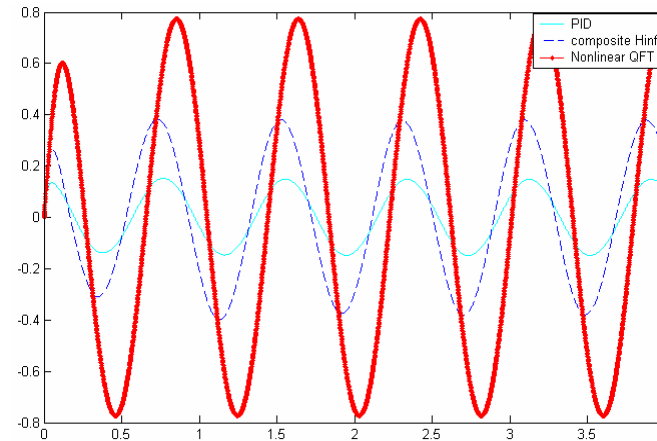




# Composite QFT Design on FJR

- Performance:
  - ✓ Error is too large

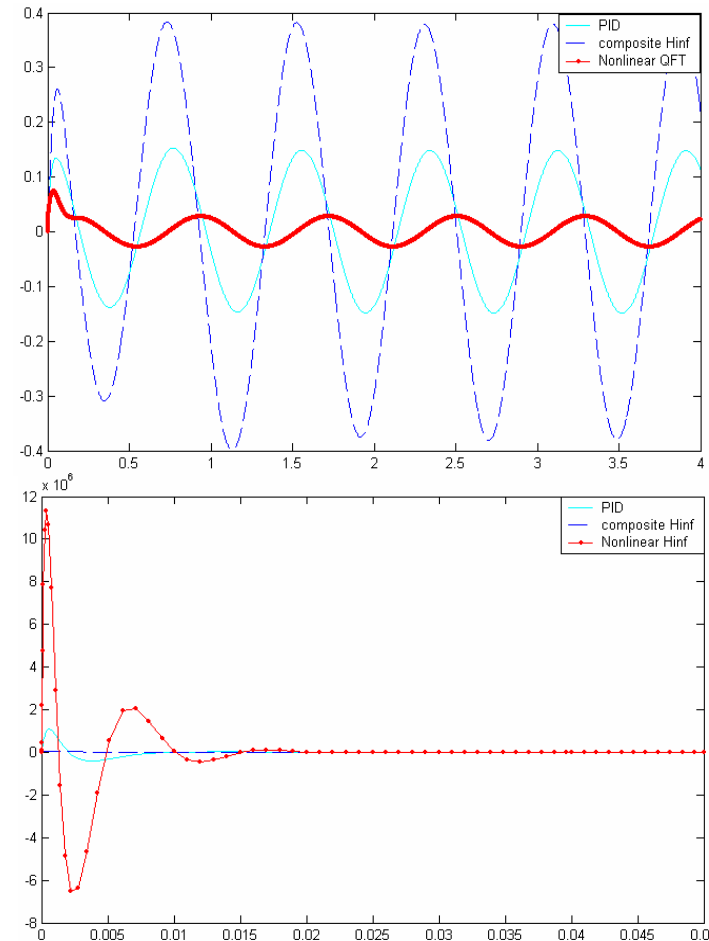
- Modify  $F(s)$ 
  - ✓  $F_2(s)=0.9716$



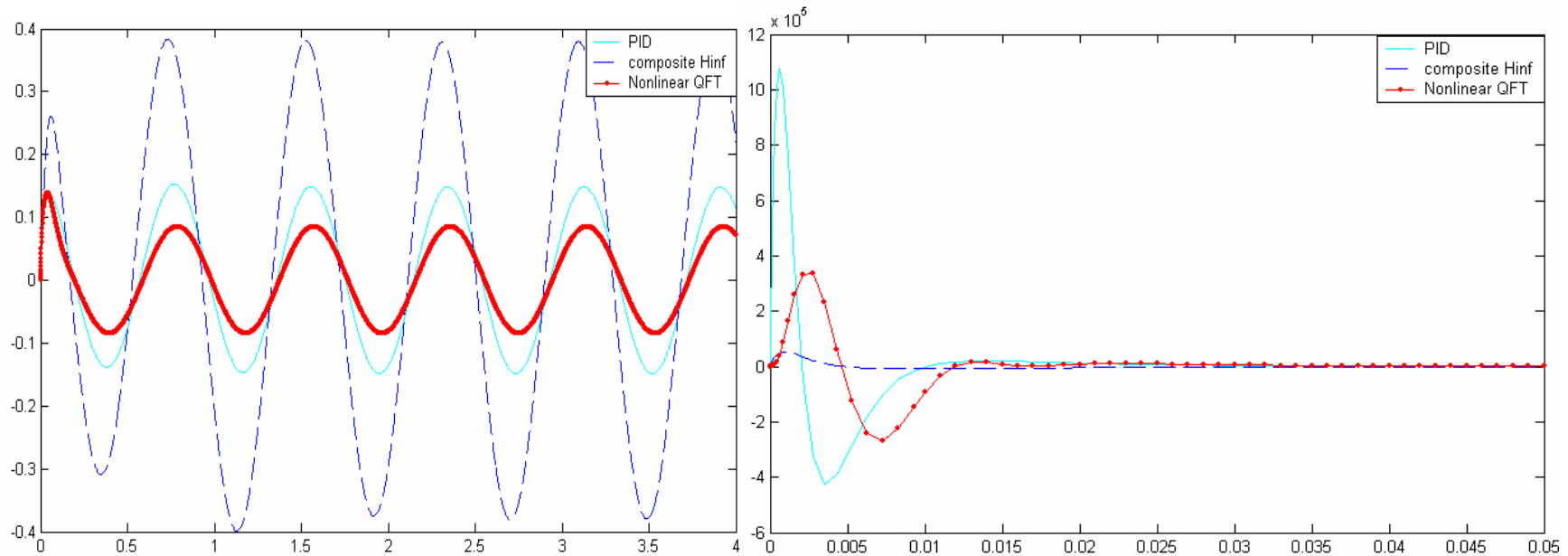
# Composite QFT Design on FJR

- Performance:
  - ✓ Good Tracking
  - ✓ Control Effort too large
- Modify  $F(s)$  : A Tradeoff

$$F_3(s) = \frac{35251}{s^2 + 305s + 3.23 \times 10^4}$$

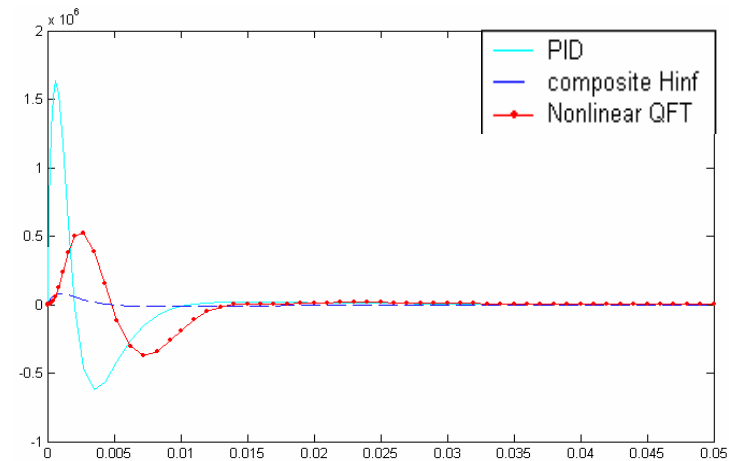
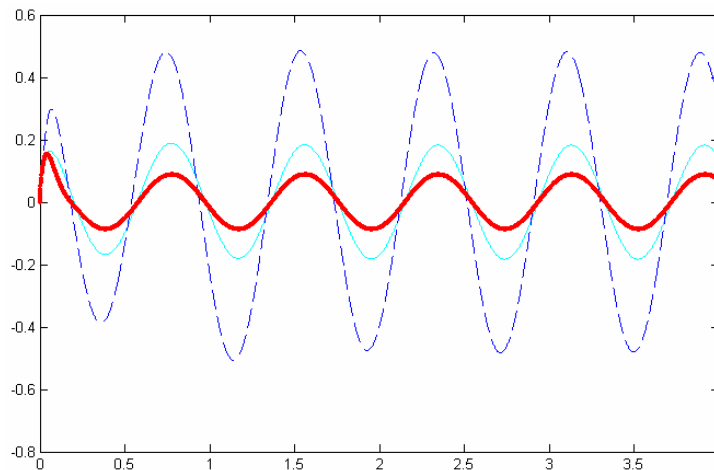


# Simulation Results



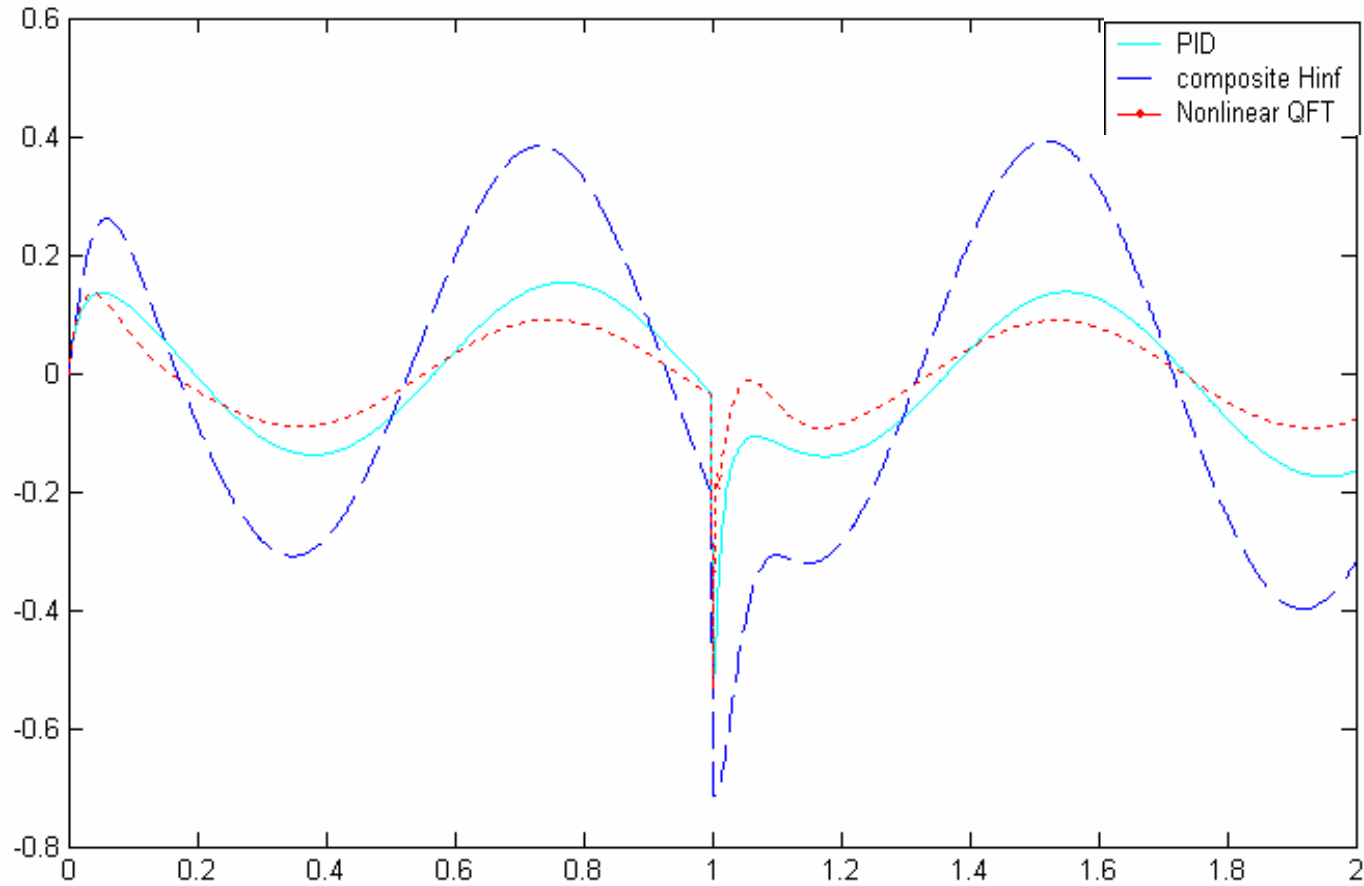
Control method	$\ e\ _2$	$\ e\ _\infty$	$\ \Delta u\ _\infty$
<i>PID</i>	0.21	0.15	$1.07 \times 10^6$
<i>Composite <math>H_\infty</math></i>	0.52	0.40	$0.54 \times 10^5$
<i>Nonlinear QFT</i>	0.12	0.14	$0.04 \times 10^5$

# Simulations: 25% Parameter Perturbation



Control method	$\ e\ _2$	$\ e\ _\infty$	$\ \Delta u\ _\infty$
<i>PID</i>	0.26	0.19	$1.63 \times 10^6$
<i>Composite <math>H_\infty</math></i>	0.67	0.50	$8.16 \times 10^4$
<i>Nonlinear QFT</i>	0.13	0.15	$5.19 \times 10^5$

# Simulations: Disturbance Rejection



# Conclusions

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- Implementation of Nonlinear QFT for FJR
  - ✓ Equivalent LTI Family
  - ✓ Composite Controller Structure
- Simulation Analysis
  - ✓ Performance Tradeoffs
  - ✓ Performance Comparison
  - ✓ Robustness
  - ✓ Disturbance Rejection
- Drawbacks
  - ✓ Skill of Designer
  - ✓ Sensitive to Noise



# Q & A

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# Thank You

