IROS 2004, Sendai, Japan

A Robust Linear Controller for Flexible Joint Manipulators

H.D. Taghirad and M. Khosravi

K.N. Toosi University of Technology,
Department of Electrical Engineering,
Advanced Robotics and Automated Systems,
Tehran, IRAN.

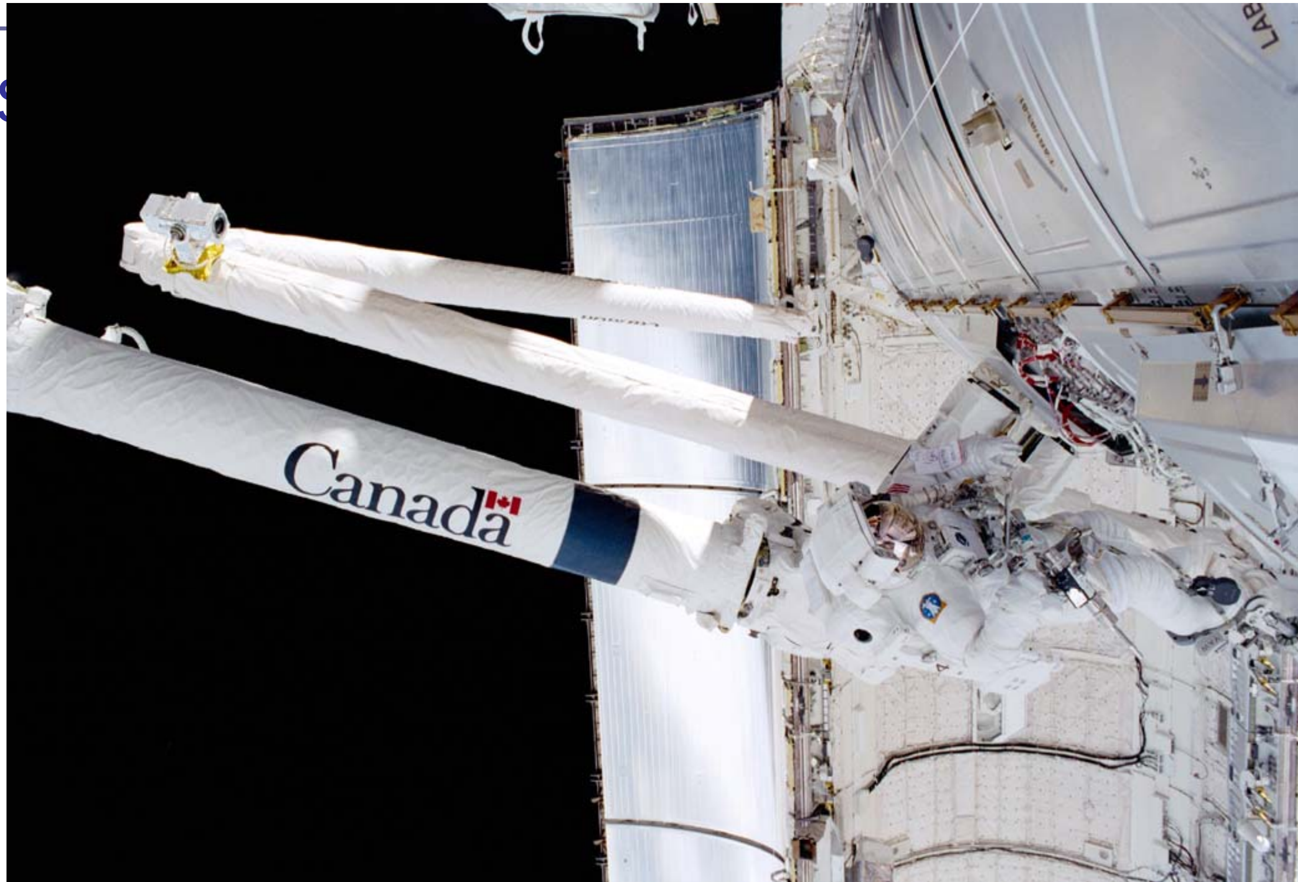
Outline

- Introduction
- Background
- FJR Modeling
- Controller Synthesis
  - ✓ Robust PID Control For Rigid Robot
  - ✓ Proposed Controller For Flexible System
- Stability Analysis
- Simulations
- Conclusions

Introduction

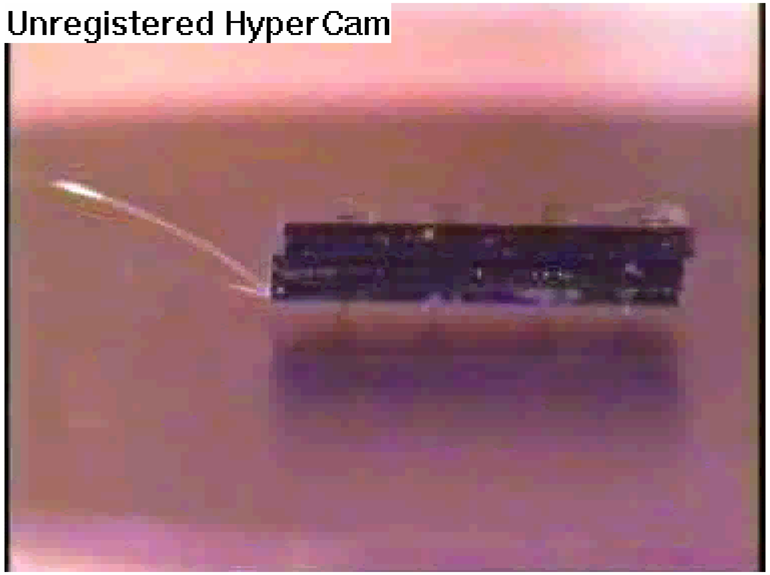
- FJR Applications
  - ✓ Space Robotics
  - ✓ Micro Robots
  - ✓ Dextereous Robotic Hand
  - ✓ Industrial Robots with Harmonic Drive

Space Robotics

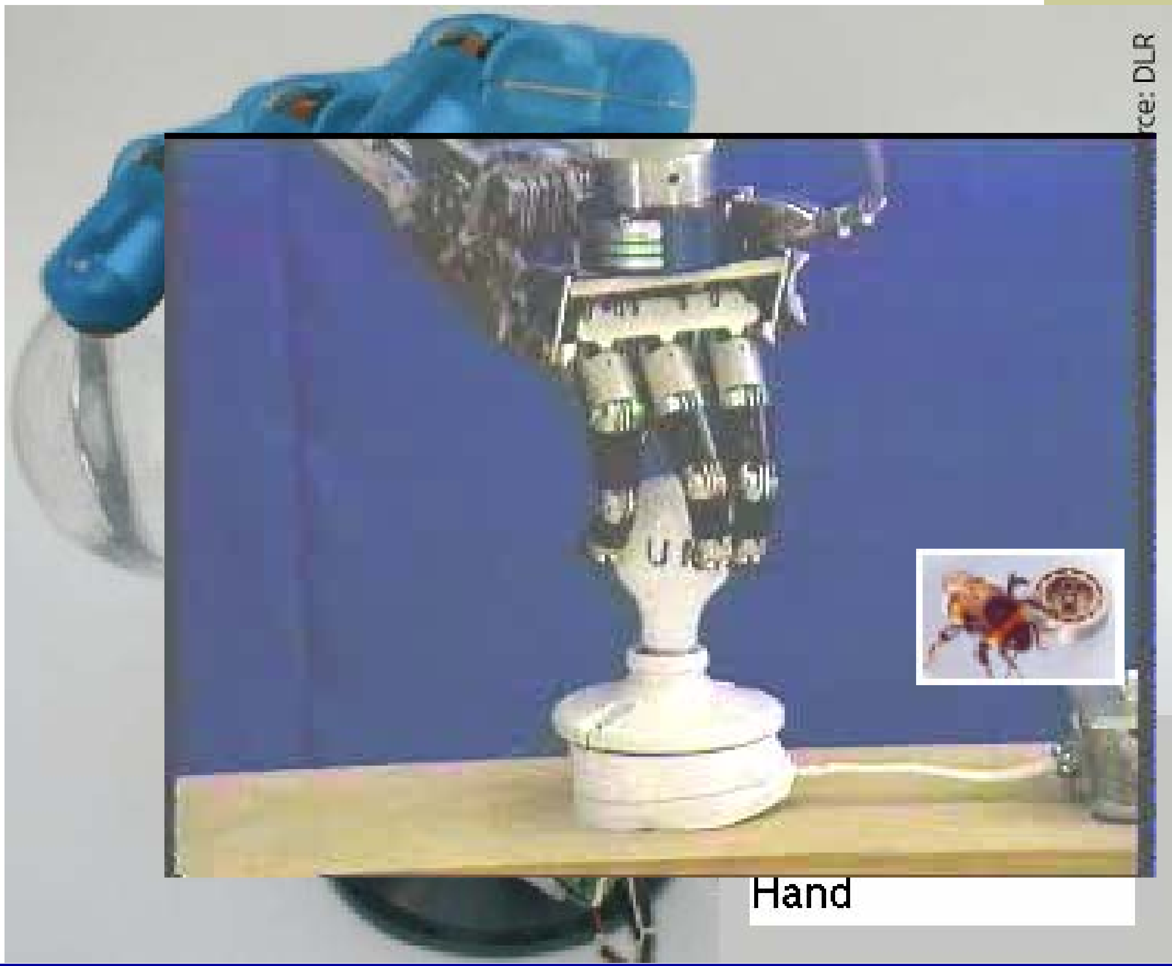


Micro Robots

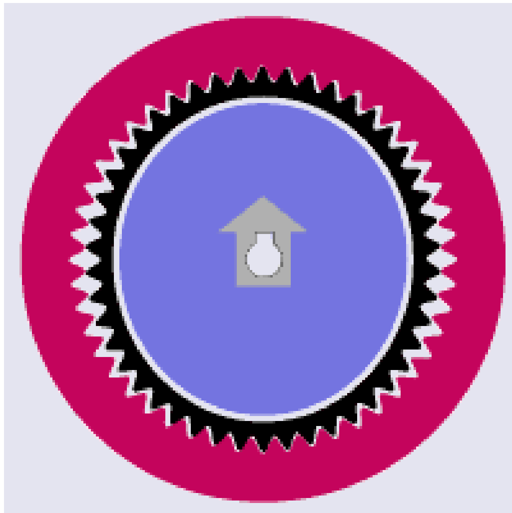
- MIT Micro Robot



Robotic Hand



Industrial Robots with Harmonic Drive



Background

• Marino,	1984	• Reshmin,	2000
• Spong,	1986	• Schaffer,	2000
• Kokotovic,	1987	• Thummel,	2001
• Ghorbel,	1991	• Bakhshi,	2003
• Khorasani,	1993	• Shaterian	2004
• Elmaraghi	1999	• Rahimi	2004

FJR Modeling

- Rigid Model:

$$\begin{cases} \tau = M(q)\ddot{q} + N(q,\dot{q}) \\ N(q,\dot{q}) = V_m(q,\dot{q})\dot{q} + G(q) + F_d\dot{q} + F_s(\dot{q}) + T_d \end{cases}$$

- Dynamics Features
  - ✓ Mass matrix pos. def.
  - ✓ $\dot{M}(q) - 2V_m(q,\dot{q})$ is skew symmetric
  - ✓ Friction terms are dissipative

$$y^T(F_d y + F_s(y)) \geq y^T F_d y \geq k_f \|y\|^2$$

  - ✓ Revolute joint property

$$L(q + 2k\pi, \dot{q}) = L(q,\dot{q})$$

  - ✓ boundedness

$$m_1 I \leq M(q) \leq m_2 I$$

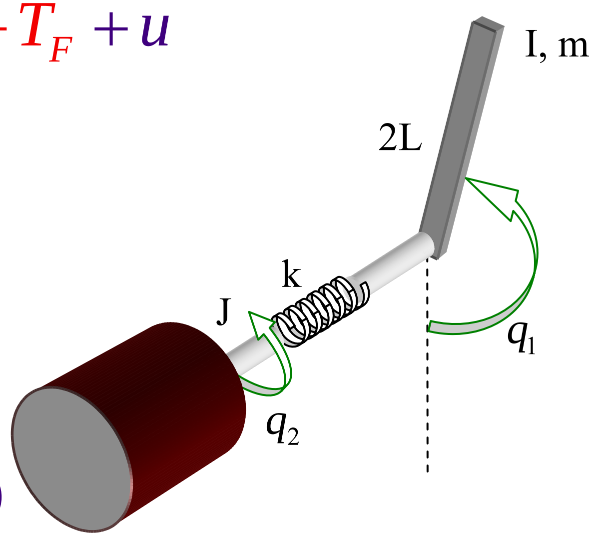
$$V_m(q,\dot{q}) \leq \xi_c \|\dot{q}\|$$

$$\vdots$$

FJR Modeling

- Flexible Model:

$$\begin{cases} M(q_1)\ddot{q}_1 + N(q_1,\dot{q}_1) = K(q_2 - q_1) \\ J\ddot{q}_2 = K(q_2 - q_1) - D\dot{q}_2 + T_F + u \end{cases}$$



- Where

  q_1 = Link Position

  q_2 = Actuator Position

  ✓ $N(q_1,\dot{q}_1) = V_m(q_1,\dot{q}_1)\dot{q}_1 + G(q_1) + F_d\dot{q}_1 + F_s(\dot{q}_1) + T_d$

Robust PID Control of Rigid Robots

- Rigid Model $q_1 = q_2$

$$M_t(q)\ddot{q} + N_t(q,\dot{q}) = u_r$$

- Apply PID Control

$$u_r = K_V\dot{e} + K_P e + K_I \int_0^t e(s)ds = Kx$$

$$\begin{cases} e = q_d - q \\ K = [K_I, K_P, K_V] \end{cases}$$

- Closed Loop System

$$\dot{x} = Ax + B\Delta A$$

Robust PID Control of Rigid Robots

Define:

$$\xi_1 = \gamma - \lambda_1 \beta_3 - \lambda_2 \bar{m} - \alpha_2{}^{-1} \lambda_1 \beta_1$$

In which,

$$\gamma = \min\{\alpha_2 k_I, \alpha_1 k_P - \alpha_2 k_V - k_I, k_V\}$$

- Theorem 1:
  System is stable of the form of UUB, if ξ_1 is chosen large enough.

Proposed Controller For Flexible System

- Apply Composite PID Control:

$$u = u_r + K_d(\dot{q}_1 - \dot{q}_2)$$

Where,

$$z = K(q_2 - q_1)$$

- The closed loop system has the form of

$$\begin{cases} M(q)\ddot{q}_1 + N(q_1, \dot{q}_1) = z \\ \varepsilon^2 J\ddot{z} + \varepsilon K_2 z + K_1 z = K_1(u_r - J\ddot{q}_1) \end{cases}$$

Which is in the form of Singular perturbation.

Stability Analysis

- Slow Subsystem:

$$(M(\bar{q}_1)+J)\ddot{\bar{q}}_1+N_t(\bar{q}_1,\dot{\bar{q}}_1)=\bar{u}_r$$

- Fast Subsystem: $\tau = t/\varepsilon$

$$J\frac{d^2\eta}{d\tau^2}+K_2\frac{d\eta}{d\tau}+K_1\eta=0$$

- Apply Thikhonov Theorem:

$$\begin{cases} z(t)=\bar{z}(t)+\eta(t)+O(\varepsilon) \\ q_1(t)=\bar{q}_1(t)+O(\varepsilon) \end{cases}$$

Stability Analysis

- FJR Dynamics:

$$\begin{cases} (M(q_1)+J)\ddot{q}_1 + N_t(q_1,\dot{q}_1) = u_r + \eta(t/\varepsilon) \\ J\dfrac{d^2\eta}{d\tau^2} + K_2\dfrac{d\eta}{d\tau} + K_1\eta = 0 \end{cases}$$

- Complete Closed Loop System:

$$\begin{cases} \dot{x} = Ax + B\Delta A + C\left[I, 0\right]y \\ \dot{y} = \tilde{A}y \end{cases} \qquad \begin{aligned} x &= [\textstyle\int_0^t e(s)^T ds,\ e^T,\ \dot{e}^T]^T \\ y &= [\eta^T,\ \dot{\eta}^T]^T \end{aligned}$$

Stability Analysis

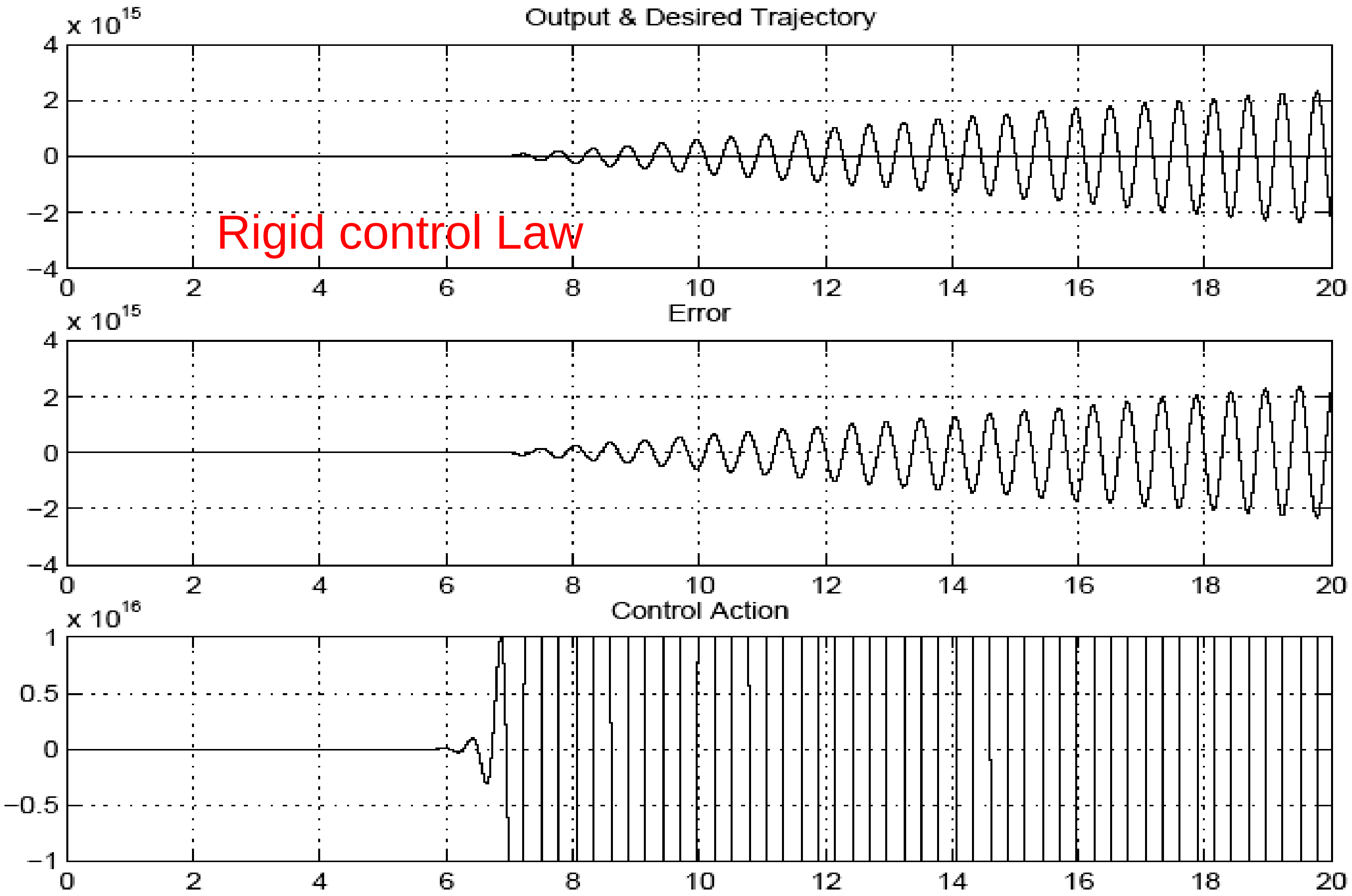
- Theorem 2

There is a positive definite matrix K_d stabilizing the closed loop system $\dot{y} = \tilde{A}y$ asymptotically.

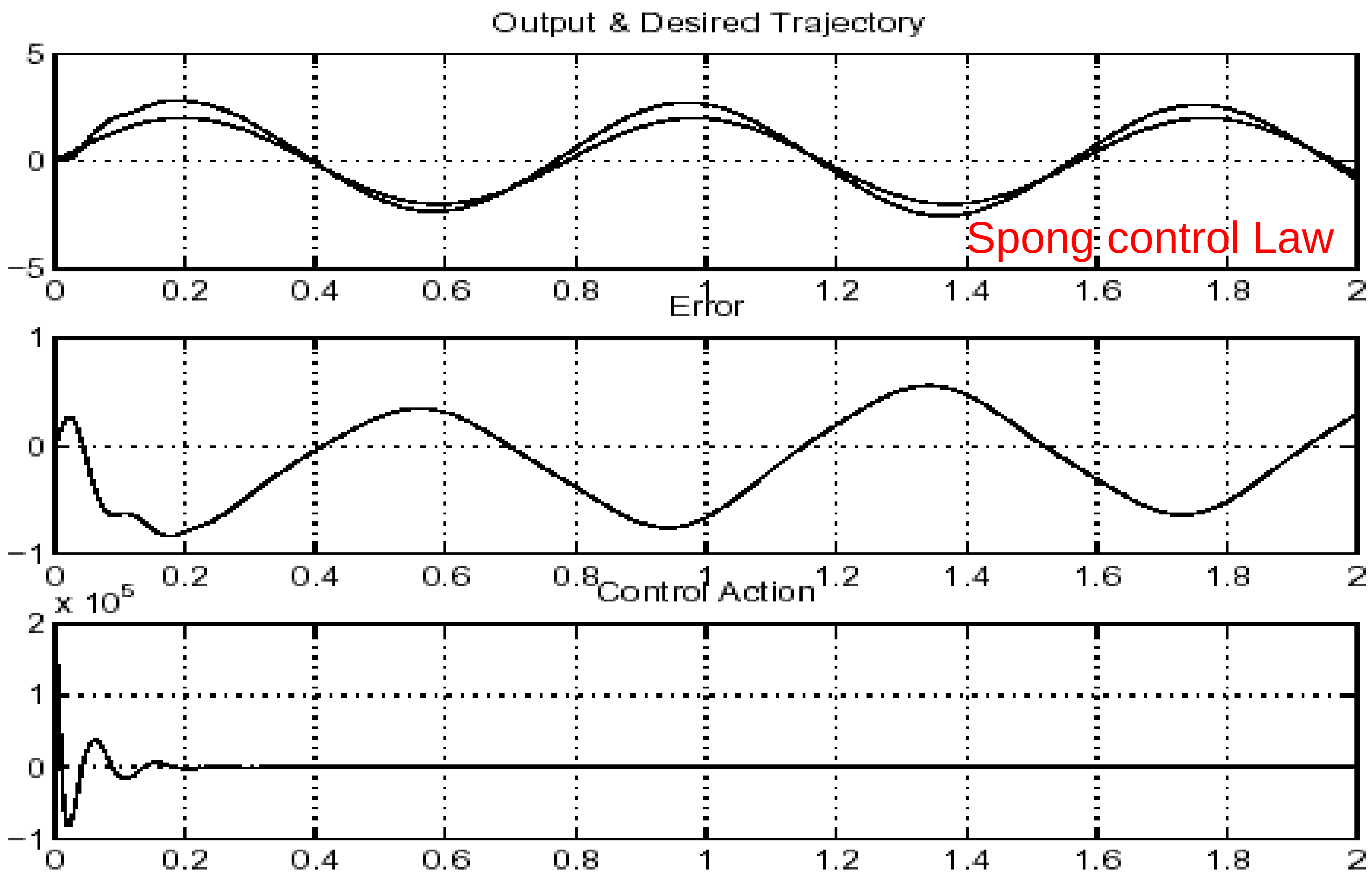
- Theorem 3

The Complete Closed loop system is UUB stable if K_d and ξ_1 are chosen Sufficiently large.

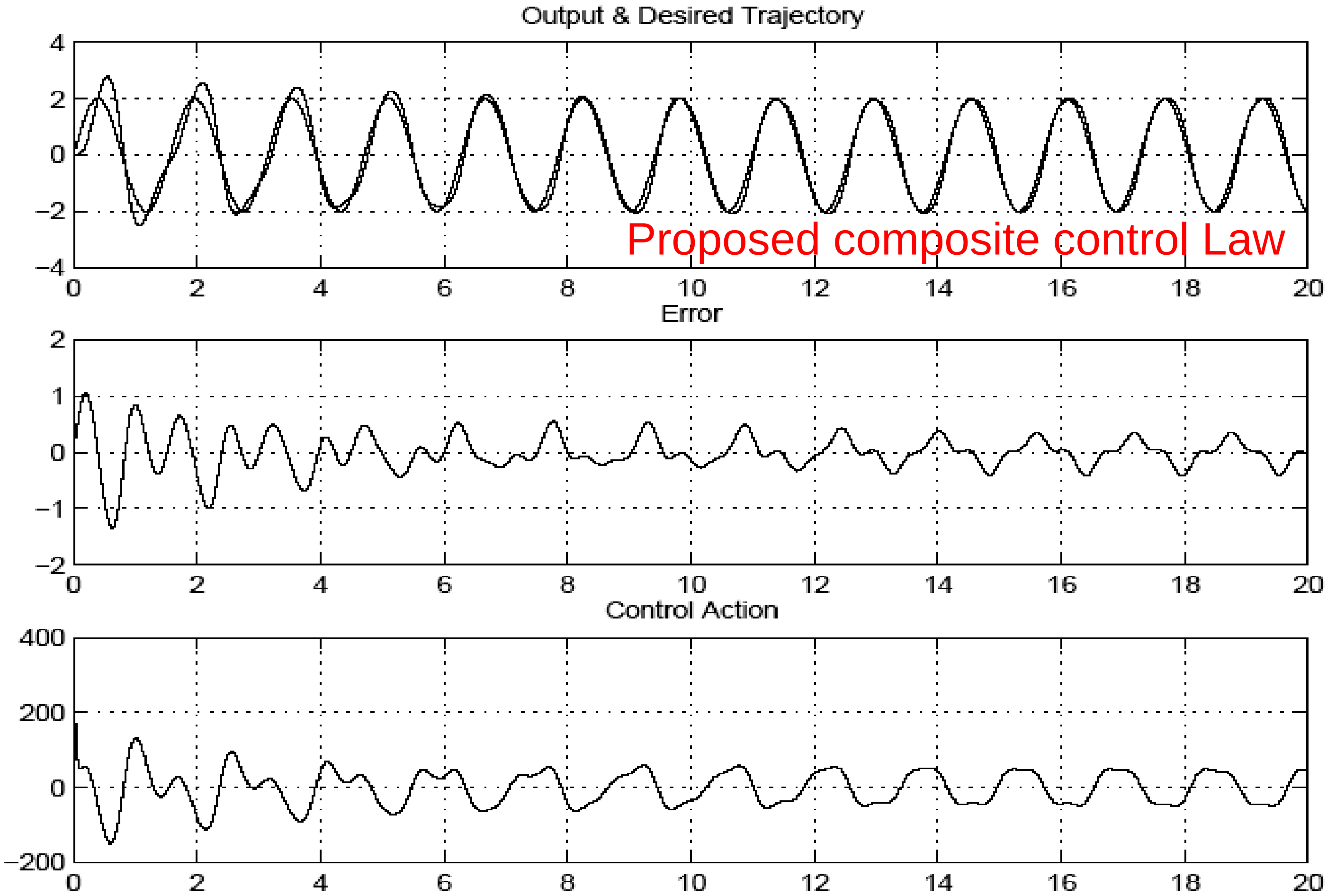
Simulations



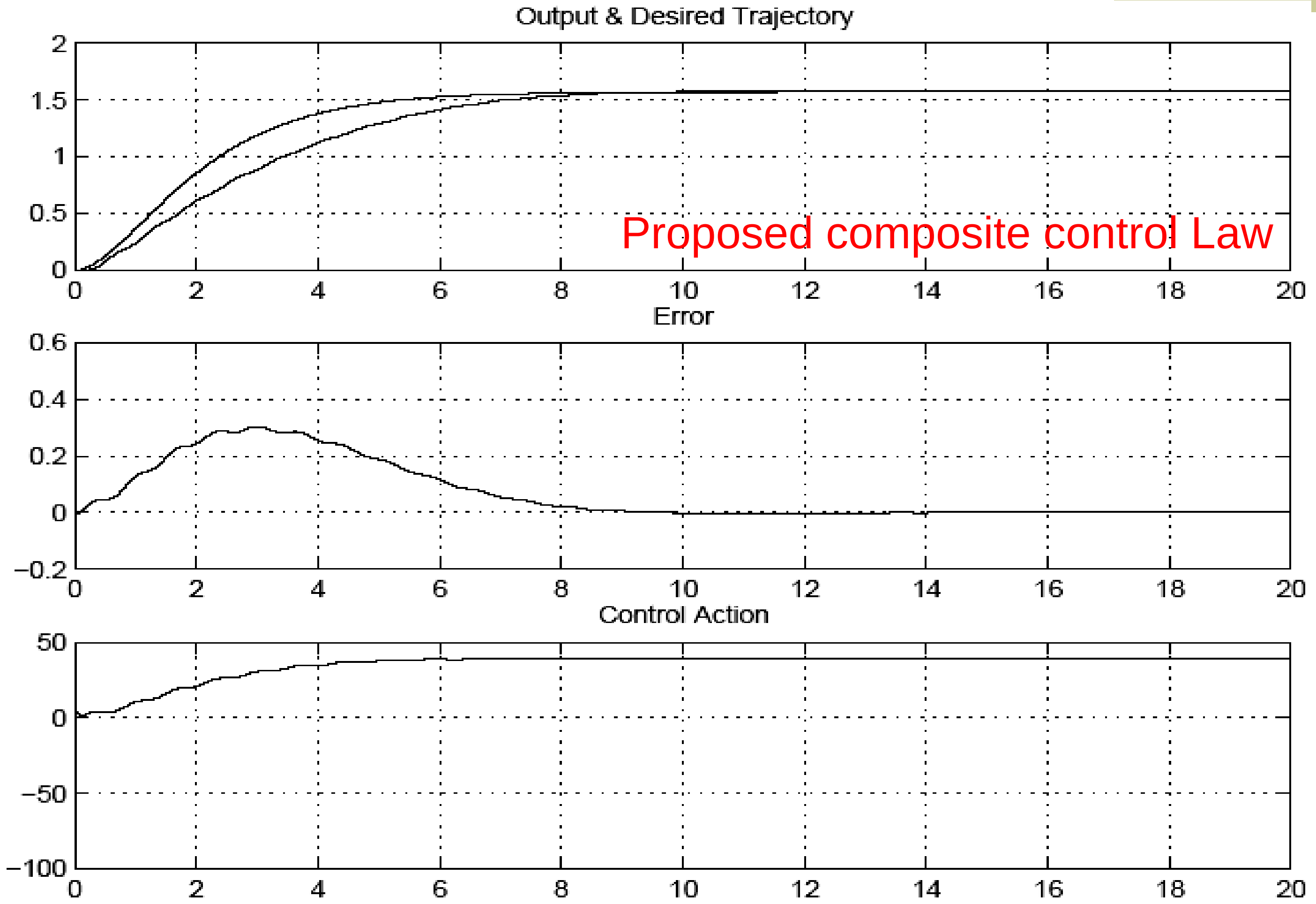
Simulations



Simulations



Simulations



Conclusions

- Implementation of robust PID control for FJR
- Stability analysis of the overall closed loop system
- Totally linear controller for slow and fast dynamics
- Comparison of results
  - ✓ Stability properties
  - ✓ Performance compared to more complicated controllers

Thank You