On The Control of the KNTU CDRPM:
A Cable Driven Redundant Parallel Manipulator

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Abstract—This paper is devoted to the control of a cable driven redundant parallel manipulator, which is a challenging problem due to the optimal resolution of its inherent redundancy. Additionally to complicated forward kinematics, having a wide workspace makes it difficult to directly measure the pose of the end-effector. The goal of the controller is trajectory tracking in a large and singular free workspace, and to guarantee that the cables are always under tension. A control topology is proposed in this paper which is capable to fulfill the stringent positioning requirements for these type of manipulators. Closed–loop performance of various control topologies are compared by simulation of the closed–loop dynamics of the KNTU CDRPM, while the equations of parallel manipulator dynamics are implicit in structure and only special integration routines can be used for their integration. It is shown that the proposed joint space controller is capable to satisfy the required tracking performance, despite the inherent limitation of task space pose measurement.

I. INTRODUCTION

Increasing performance requirements necessitates design of new types of manipulators working in a larger dexterous workspace with higher accelerations. Parallel manipulators can generally perform better than serial manipulators in terms of stringent stiffness and acceleration requirements [1]. In parallel manipulators, each actuated leg has to carry only a part of the payload, and therefore, the robot can handle heavy load to weight ratios. However, limited workspace and existence of many singular regions inside the workspace of a typical parallel manipulator, limits the use of parallel manipulators in various applications. In the case of cable driven redundant parallel manipulators (CDRPM), the conventional linear actuators of a parallel manipulators are replaced with electrical powered cable drivers. This novel engineering design idea leads immediately to a wider workspace, and higher accelerations of the moving platform due to the fact of using light moving parts [2]. The payoffs of these significant advantages of CDRPM is some challenging topics of research which has attracted the robotics community attention. Generally, forward kinematics of a parallel manipulator like CDRPM is very complicated and difficult to solve [3]. Cables are sagged under compression forces [4], and therefore, to achieve tension forces on cables in a dextrous workspace, moving platform must be designed over–constrained [5]. In this case \( m = n + 2 \) cables are used in order to move the redundant actuated end–effector in an \( n \)–dimensional space. Redundancy resolution is needed to assure tension force existence along each cable, however, this is usually computationally expensive [6]. The control algorithms developed for serial counterparts to parallel manipulators with redundant actuation in [7]. A controller with positive tension for six degrees of freedom cable suspended robot is designed in [8]. A target tracking manipulation method is developed in [9], which can control position and contact force of robotic manipulators with an acceptable error without requiring the solution for both direct kinematics and inverse kinematics. Among the many control topologies reported in the literature, the control of redundantly actuated parallel manipulators has been considered by fewer researchers. However, only a few of the proposed topologies can be implemented in a cable driven redundant parallel manipulator.

In this paper different control topologies examined for possible implementation on KNTU CDRPM are introduced. KNTU CDRPM uses a novel design to achieve high stiffness, accurate positioning and high-speed maneuvers. In here, position control of this parallel manipulator is studied in detail. The inverse kinematics is derived first. The traditional Newton–Euler formulation is used for the dynamic modeling of this manipulator [10]. Two control topologies based on inverse dynamics in the workspace and joint space coordinates are analyzed in this paper. It is demonstrated that the inverse dynamics control in the joint space is more suitable for possible implementation on the cable driven redundant parallel manipulator. Finally, a new control topology is proposed that can be used for these type of manipulators. The proposed controller structure is not only preserving the advantages of joint space controller, but also promising fully tension forces on the cables, in a more trusted fashion.

Fig. 1. The KNTU CDRPM, a perspective view
II. KINEMATICS AND DYNAMICS

A. Kinematics

The KNTU Cable Driven Redundant Parallel Manipulator is illustrated in figure 1. This figure shows a spatial six degrees of freedom manipulator with two degrees of redundancy. This robot has eight identical cable limbs. The cable driven limbs are modeled as spherical-prismatic-spherical (SPS) joints, for cables can only bear tension force and not radial or bending force. Two cartesian coordinate systems $A(x, y, z)$ and $B(u, v, w)$ are attached to the fixed base and the moving platform. Points $A_1, A_2, \ldots, A_8$ lie on the fixed cubic frame and $B_1, B_2, \ldots, B_8$ lie on the moving platform. The origin $O$ of the fixed coordinate system is located at the centroid of the cubic frame. Similarly, the origin $G$ of the moving coordinate system is located at centroid of the cubic moving platform. The transformation from the moving platform to the fixed base can be described by a position vector $\mathbf{g}$ and a $3 \times 3$ rotation matrix $A R_B$. Consider $a_i$ and $b_i$ denote the position vectors of points $A_i$ and $B_i$ in the coordinate system $A$ and $B$, respectively. Although in the analysis of the KNTU CDRPM, all the attachment points are considered to be arbitrary, the geometric and inertial parameters given in table I are used in the simulations. Similar to other parallel manipulator, CDRPM has a complicated forward kinematic solution [3]. However, the inverse kinematic analysis is sufficient for dynamic modeling. As illustrated in figure 1, the $B_i$ points lie at the vertexes of the cube. For inverse kinematic analysis of the cable driven parallel manipulator, it is assumed that the position and orientation of the moving platform $x = [x_G, y_G, z_G]^T, A R_B$ are given and the problem is to find the joint variable of the CDRPM, $L = [L_1, L_2, \ldots, L_8]^T$. From the geometry of the manipulator as illustrated in figure 2 the following vector loops can be derived:

$$A A_i B_i + A a_i = A \mathbf{g} + E_i$$

in which, the vectors $\mathbf{g}, E_i$, and $a_i$ are illustrated in figure 2. The length of the $i$th limb is obtained through taking the dot product of the vector $A_i B_i$ with itself. Therefore, for $i = 1, 2, \ldots, 8$:

$$L_i = \left\{ [g + E_i - a_i]^T [g + E_i - a_i] \right\}^{1/2}.$$  

B. Dynamics

Newton-Euler method is used for dynamic modeling of CDRPM [10]. According to acceleration of rotating velocity vector [11], [10], the Newton-Euler equations for varying mass cable results into:

$$F_{Bi} = -\frac{1}{2} \rho L_i^2 [L_i \omega_i \times S_i + \omega_i \times S_i + \omega_i \times (\omega_i \times S_i)]$$

$$= -\frac{\rho}{2} (L_i^2 + L_i \hat{L}_i) \hat{S}_i + F_{Ai}$$  (3)

Where $F_{Bi}, F_{Ai}, L_i, \hat{S}_i, \omega_i$ and $\hat{\omega}_i$ are resultant acting force on the each moving attachment point, acting forces on the $A_i$ fixed joint, cable linear velocity along its straight, the unit vector on ith cable straight as shown in figure 2, the ith cable angular velocity about the fixed attachment point and the ith cable angular acceleration about the fixed attachment point respectively. By using light weight cables such as the ones used in this manipulator, the gravity force effects on the cables can be ignored compared to the dynamic induced forces [12]. The cable’s tension force applied by cable driver unit, $F_{Ai}^\tau$, can be represented by:

$$F_{Ai}^\tau = -\tau$$  (4)

Relations between actuator forces and the end-effector affected forces had been studied in cable-affected forces. Writing the Newton-Euler equations for moving platform describes the relation between forces, torques and acceleration of the moving platform as following:

$$M \ddot{\mathbf{x}} = F_D + G + \sum_{i=1}^{n} F_{Bi}$$  (5)

$$I_G \ddot{\theta} = \tau_D - \sum_{i=1}^{n} E_i \times F_{Bi}$$  (6)

In which, $M$ and $I_G$ are moving platform’s mass and moment of inertia and $n$ is number of the cables. $G$ is effect of gravity force on the end-effector, $F_D$ and $\tau_D$ are disturbance forces and torques effects on moving platform with respect
Inverse Dynamics

FIDC

\[ f_d(x, \dot{x}, \ddot{x}, \tau) = 0 \]  \hspace{1cm} (7)

Where, \( \vec{3}_D \) is the vector of disturbance forces and moments. The governing motion equations of the manipulator can be implemented for dynamic simulation of the system. For dynamic simulation, it is assumed that the actuator forces \( \tau(t) \), are given and the manipulator motion trajectory \( x(t) \), is needed to be determined. As it is explained in sections III-B and III-A due to the implicit form of the dynamic equation, special integration

\[ \tau > \tau_{min} \]  \hspace{1cm} (8)

\[ \tau = J^T \vec{F} \]  \hspace{1cm} (9)

The first non-equality constraint assures that the cables are always under tension. The second equality constraint assures that the joint space forces, \( \tau \) are the right projection of the cartesian force \( \vec{F} \). Other optimization techniques can be used to find the actuator forces projected from, \( \vec{F} \) which can minimize a user defined cost function [6].
The block diagram of IDC in the workspace coordinate is given in figure 3. The minimum norm solution, with the optimal condition is used as the redundancy resolution method. The controller used in this simulation is a decentralized PD controller, in which the gains are tuned such that the required tracking performance is achieved. The gains are given in table II. A linear trajectory with parabolic blends is considered in this simulation. As illustrated in figure 3 the vector force, \( \mathbf{F} \), in the workspace coordinate is obtained by equation 10.

\[
\mathbf{F} = \mathbf{F}_{PD} + \mathbf{F}_{IDC}
\]

Where \( \mathbf{F}_{PD} \) is the created vector force by PD controller \( \mathbf{F}_{PD} = K_p e(t) + K_v \dot{e}(t) \), where \( e(t) = x_d(t) - x(t) \) is the trajectory tracking error and \( K_p, K_v \) are same appropriate position and velocity gain matrix, whose values are given in table II. \( \mathbf{F}_{IDC} \) is the generated vector force by IDC. Inverse dynamics generated force, preserves the end-effector’s current state of acceleration and obtains required force in the workspace coordinate in the form of a feedback linearization:

\[
\mathbf{F}_{IDC} = \hat{M} \ddot{x}_d + \hat{G}
\]

Where, \( \hat{M} \), \( \hat{x}_d \), \( \hat{G} \) are inertia and gravity computed forces of the end-effector represented in the task coordinate. Time varying inertia and gravity forces of the cables are neglected in the controller because of high real time process costs and very low contribution in the amount of controller forces. \( \mathbf{F}_{IDC} \) is a 6x1 wrench vector composed of the computed force and torque. There is no term containing \( \ddot{x} \) in this equation, and this is due to the fact that no damping effect is modeled on the end-effector. Also, a proportional derivative controller can effectively reduce the tracking errors of the end-effector in the presence of disturbance force/torques. Note that due to the insignificance of the cables’ inertia terms, these terms are ignored in the feedback linearization routine [10]. Hence, the resultant control forces acting on the system are as follows:

\[
\hat{M} \ddot{x} = \mathbf{F}_D + \mathbf{F}_{PD} + \mathbf{F}_{IDC} + \hat{G}
\]

By canceling the nonlinear terms through the proposed feedback linearization method in the workspace, it is expected that a decentralized PD controller would result into an acceptable control performance, which is verified in the simulation results. The tracking performance of the CDRPM is illustrated in figures 4 and 5. As seen in the figure 5, this control topology is capable of reducing the tracking errors less then 0.4 mm in position and less than 0.1° in orientation. The actuator force of the CDRPM is shown in figures 6. As seen in this figure the simple redundancy resolution technique proposed here can efficiently distribute the cartesian force into actuator forces. However, constrained optimization routine is proposed for redundancy resolution to guarantee that the cables always remain in tension for all configuration of CDRPM maneuvers. However, the proposed topology suffers from two important implementation limitations. Firstly, the position and orientation of the platform have to be measured during the control process. Accurate measurement of the position and orientation of the platform is very difficult and expensive [18]. Moreover, it is not desirable to obtain the pose of the platform using direct kinematics, due to the complexity of calculations, [19]. Thus, IDC in joint space is advised to be implemented in this application [4].

**B. IDC in the Joint space Coordinate**

The control scheme is shown in figure 7. As seen in this figure, in this topology the actuators length \( L \) are measured and its time derivative \( \dot{L} \) are either measured or estimated. Let \( L_d, \dot{L}_d \) denote the desired cable length and its velocity which can be easily obtained through computing the inverse kinematics. In this proposed topology the PD controller is
designed in joint space, and the control efforts are directly applied through the cable driver units. This topology leads to various benefits; the resulting actuator torques can be easily studied in detail, where, the limitation of the driver’s power boundaries, specially upper limit of actuators, can be easily implemented. In this structure the control law is as follows:

\[ \tau = \tau_{PD} + \tau_{IDC} \]  

(13)

in which, \( \tau \) is an 8x1 tension force vector along each cable, and \( \tau_{PD} \) is part of the tension force in the joint space coordinate that is provided by PD controller by \( \tau_{PD} = K_p e + K_v \dot{e} \), in which, the values for \( K_p \) and \( K_v \) are presented in table II. These gains are tuned such that the required tracking performance is achieved. Let \( e(t) = L_d - L \) and \( \dot{e}(t) = \dot{L}_d - \dot{L} \) denote the error of actual cable length to that of the desired one and its derivative. Force optimization unit distributes the computed force vector \( \mathbf{F}_{IDC} \) along the actuator torques, \( \tau_{IDC} \). The simulation results are shown in figures 8 and 10. As seen in figure 10, this control topology is capable of reducing the tracking errors less than 0.2 mm in position and less than 0.02° in orientation.

The performance of the two controllers are compared and the result of comparison is given in table II. As it is shown in the third and forth rows of this table, the obtained two-norm and infinity-norm of the tracking errors are admissible for both controllers. Furthermore, positive amounts of minimum actuator forces \( \tau_{min} \) reveals that cables remains always under tension, and moreover, considering maximum actuators forces \( \tau_{max} \), it is obvious that none of the actuators reach the saturation limited boundary 1000 \( N \). Therefore, both control topologies satisfy the required tracking performance of the manipulator, despite the actuator limitations.

Note that in the joint space topology, PD controller generates part of force vector in the joint space coordinate, however, computed force by IDC is in the workspace coordinate. Thus, we need optimization routine to project force vector in the workspace into the joint space. As shown in figure 11 the actuator forces are all positive tension. If this error becomes large, the amount of vector force generated by PD controller, can be greater than that of the force vector computed by IDC. This event influences the redundancy resolution. Thus, positive tension forces can not be assured by this configuration. In order to remedy this problem, a new controller topology is proposed in this paper and illustrated in figure 12.

In this structure actuator forces are transformed from joint space into workspace by \( \mathbf{F} = \mathbf{J}^T \tau \). Hence, the previous redundancy resolution method can be employed in this structure similar to that elaborated in subsection III-A. This controller structure not only preserves the advantages of joint space controller, but also guarantees fully tension forces on the cables, in a more trusted fashion.

![Fig. 8. KNTU CDRPM cable length with IDC in the joint space.](image)

![Fig. 9. The cable length error with IDC in the joint space.](image)

![Fig. 10. The tracking error with IDC in the joint space.](image)

<table>
<thead>
<tr>
<th>TABLE II</th>
<th>CONTROLLER PARAMETERS OF THE KNTU CDRPM</th>
</tr>
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<tbody>
<tr>
<td>Parameters</td>
<td>IDC in Workspace</td>
</tr>
<tr>
<td>( K_p )</td>
<td>( 10^5 \times 1_{6\times 6} )</td>
</tr>
<tr>
<td>( K_v )</td>
<td>( 10^5 \times 1_{6\times 6} )</td>
</tr>
<tr>
<td>( 10^3 \cdot</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.4, 0.9, 0.2</td>
</tr>
<tr>
<td>( 10^3 \cdot</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.10, 0.20, 0.05</td>
</tr>
<tr>
<td>( \tau_{min}(N) )</td>
<td>35, 10, 3, 3,</td>
</tr>
<tr>
<td></td>
<td>26, 3, 36, 10</td>
</tr>
<tr>
<td>( \tau_{max}(N) )</td>
<td>558, 81, 558, 42,</td>
</tr>
<tr>
<td></td>
<td>606, 47, 605, 59</td>
</tr>
</tbody>
</table>
IV. CONCLUSIONS

In this paper the control of KNTU CDRPM is studied in detail. KNTU CDRPM has two redundant actuators in order to move its end-effector with 6 degrees of freedom. These degrees of actuator redundancy is used to keep the cables always in tension. In this paper the kinematics and Jacobian formulation of the manipulator is derived, and the dynamic modeling is performed through traditional Newton–Euler method. A decentralized controller consist of an inverse dynamics controller (IDC), and a PD controller, is proposed first, which optimally generates the required workspace forces by actuator forces using a constrained optimization technique. The closed-loop performance of the manipulator is simulated in computer, and the results has been analyzed. Since the position, velocity and acceleration of the end-effector is needed to implement this algorithm and in practice, the measurement of these variables are expensive, another controller topology in the joint space is proposed. In this method neither the end-effector position nor the solution to forward kinematics is needed. It is shown that the obtainable tracking performance by this means is less 0.2mm in position and less than 0.02° in orientation, for this topology. This method is promising and can be used in real time implementation. The last investigated control topology can be carefully generalized for the other cable parallel redundant manipulators.

REFERENCES