Robust Torque Control of Harmonic Drive
Under Constrained-Motion

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Abstract—
A harmonic drive is a compact, light-weight and high-ratio torque transmission device which has almost zero backlash. Its unique performance features captures the attention of designers in many industrial applications, especially in robotics. However, the torque control of harmonic drive systems is still a challenging problem for researchers. In this paper the torque control of harmonic drive system for constrained motion is examined in detail. A nominal model for the system is obtained from experimental frequency responses of the system, and the deviation of the system from the model is encapsulated by multiplicative uncertainty. A robust torque controller is designed using this information in an $\mu$-framework, and implemented on two different setups. It is illustrated that the performance features of the closed-loop system is exceptionally good, both in time and frequency domains.

I. INTRODUCTION

Developed in 1955 primarily for aerospace applications, harmonic drives are high-ratio and compact torque transmission systems. Every harmonic drive consists of the three components illustrated in Figure 1. The wave generator is a ball bearing assembly with a rigid, elliptical inner face and a flexible outer face. The flexspline is a thin-walled, flexible cup adorned with small, external gear teeth around its rim. The circular spline is a rigid ring with internal teeth machined along a slightly larger pitch diameter than those of the flexspline. When assembled, the wave generator is nested inside the flexspline, causing the flexible circumference to adopt the elliptical profile of the wave generator, and the external teeth of the flexspline to mesh with the internal teeth on the circular spline along the major axis of the wave generator ellipse.

If properly assembled, all three components of the transmission can rotate at different but coupled velocities on the same axis. To use the harmonic drive for speed reduction, the wave generator is mounted on the electric motor shaft, and the output is conveyed either through the flexspline while the circular spline is fixed or through the flexspline while the circular spline is fixed. In the latter case, by rotation of the wave generator the zone of gear-tooth engagement is carried with the wave generator major elliptical axis. When this engagement zone is propagated 360° around the circumference of the circular spline, the flexspline which contains fewer teeth than the circular spline, will lag by that fewer number of teeth relative to the circular spline. Through this gradual and continuous engagement of slightly offset teeth, every rotation of the wave generator moves the flexspline a small angle back on the circular spline, and through this unconventional mechanism, gear ratios up to 320:1 can be achieved in a single transmission.

The harmonic drive exhibits performance features both superior and inferior to those of conventional gear transmissions. Its performance advantages include high-torque capacity, concentric geometry, lightweight and compact design, zero backlash, high efficiency, and back drivability. Harmonic drive systems suffer however, from high flexibilility, resonance vibration, friction and structural damping nonlinearities.

In many applications specifically in robotics, torque is often taken to be the control input. The physical variable being manipulated, however, is not torque but armature current in a DC motor, for instance. For harmonic drive systems the relation between output torque and input current possesses nonlinear dynamics, due to the flexibility, Coulomb friction and structural damping of the harmonic drive [13]. Therefore, it is desired to improve this input/output relation by torque feedback, and to convert the system to a torque source with a flat frequency response over a wide bandwidth.

There are two types of torque-control applications for a robot manipulator using harmonic drives in its joints. First the applications where the robot is in contact with a stiff environment, and high torques at very low velocities are re-

Fig. 1. Harmonic drive components

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quired at each joint. Simulation of this application at each joint can be studied by a constrained motion experiment. The problem of torque control of a robot joint actuator in constrained motion is addressed in this paper. This torque control can be used in high-frequency oscillation depression of a robot link as well, where the vibration of the link is modelled by a constrained motion for the actuators. The second type of applications occurs when the robot arms are moving freely, and the only torque required at each joint is to compensate for gravity, friction and link acceleration. This problem can be simulated through a free-motion case which will be addressed separately [14]. In the free-motion case the amount of torque required at each joint is very low but at much higher velocities. The different nature of these two types of applications introduce different challenging problems.

Throughout its short existence, the harmonic drive has enjoyed increasing international attention from designers as well as researchers. Russians were perhaps the first who initiated substantial research on the dynamic behavior of harmonic drives [16]. More recently Taghirad and Bélanger obtained simple and accurate models for friction, stiffness, and structural damping of harmonic drive systems and verified the performance of the simulated model with experiments in both constrained and free motion cases [13]. Tuttle performed an intensive effort to model the stiffness, positioning accuracy, gear tooth-meshing mechanism and friction of the harmonic drive [15]. Kircanski and Goldenberg have also attempted to model the harmonic drive in detail [12]. Bridges et al. [3], Kaneko et al. [10], Kazerooni [11], Hogan [9], and Chapel and Su [4] are representative of researchers who worked on the control of harmonic drive system. Bridges [3] used a very simple linear model for the system, with PD torque control. His results show some improvement in tracking error, but insufficient performance near resonant frequency. Kaneko [10], also based his analysis on a simple model of the system, but included nonlinear stiffness in the system. He then applied a feedforward loop to adjust for nonlinear stiffness and then a pure gain torque feedback to shape the performance. Kazerooni [11], considers a simple linear system for the harmonic drive, and used a sensitivity loopshaping technique to design a linear controller for the system. Hogan [9], proposed impedance control for robots with harmonic drive systems, to deal with the dynamic interaction induced in contact tasks. Chapel [4], applied $\mathcal{H}_\infty$ control design methods to the analysis and design of impedance control laws.

In this paper, a general framework to design torque controllers for harmonic drive system in constrained motion is presented. It is shown that an empirical linear model obtained from experimental frequency responses of the system, and an uncertainty characterisation of this model is enough to build a robust torque controller. The $\mathcal{H}_\infty$ framework is used for controller design, and the proposed controller is tested for two different setups. The closed-loop performance in time and frequency domain are shown to be exceptionally good.

II. EXPERIMENTAL SETUP

Two harmonic drive testing stations were used to monitor the behaviour of two different harmonic drives. A picture of the setup and its schematics are illustrated in Figure 2, in which the harmonic drive is driven by a DC motor. A positive locking system is designed such that the harmonic drive output is locked to the ground. In the first setup, a brushed DC motor from Electro-Craft model 586-501-113 is used. Its weight is 1360 grammes, with maximum rated torque of 0.15 Nm, and torque constant of 0.0543 Nm/amp. The servo amplifier is a 40 Watts Electro-Craft power amp model Max-100-115. The harmonic drive in the first setup is from RHS series of HD systems model RHS-20-100-CC-SP, with gear ratio of 100:1, and rated torque of 40 Nm. The DC motor in the second setup is a brushless Kollmorgen Inland motor model RBE-01503-A00. Its weight is 475 grammes, with maximum rated torque of 5.6 Nm, and torque constant of 0.1815 Nm/amp. The servo amplifier is a FAST Drive Kollmorgen, model FD 100/5E1. The harmonic drive is from CFS series of HD Systems, Inc. with gear ratio 160:1, and rated peak torque of 178 Nm.

In the first experimental setup, the circular spline is fixed to the ground and the output is carried by the flexspline, while in the other setup, the flexspline is fixed and the circular spline is used for output rotation. By this arrangement, the behavior of the transmission under different operating configurations can be experimented. Each setup is equipped with a tachometer to measure the motor velocity, and the output torque is measured by a Wheatstone bridge of strain gauges mounted directly on the flexspline [8]. The current applied to the DC motor is measured from the servo amplifier output. These signals were processed by several data acquisition boards and monitored by a C-30 Challenger processor executing compiled computer C codes. Moreover, Siglab [7], a DSP hardware linked to Mat-
lab, is used for frequency response analysis of the system. This hardware is capable of generating sine-sweep, random, and chirp function inputs to the system, and analyse the output and produce online frequency response estimates of the system.

III. SYSTEM MODEL AND ITS UNCERTAINTY

A complete model of the system was derived in [13]. To capture the system dynamics accurately, it is necessary to consider nonlinear models for friction and structural damping. However, for the purpose of control, a linear model for the system will be used for the synthesis. An empirical method to find this nominal model is to perform a series of experimental frequency response on the system, with different input amplitudes, and to find the best fit through them. By this method, not only the empirical nominal model of the system (without need of any linearization) will be determined, but also variations in the frequency response of system, due to the nonlinearities, will be encapsulated with an uncertainty representation. Using Siglab-generated sine-sweep and random inputs with different amplitudes on each experimental setup, a set of frequency response estimates for the system is generated. Applying an iterative Gauss-Newton routine on one of the frequency response estimates, a transfer function is obtained which minimizes the weighted least-squares error between the experimental frequency response and the model. We call this transfer function the “Nominal Model” of the system (illustrated in Figures 3 and 4).

Moreover, the variation of each frequency response estimate from the nominal model can be encapsulated by a multiplicative uncertainty. Assuming that the nominal plant transfer function is \( P_0(s) \), define \( \mathcal{P} \) as the family of possible models of the system which includes all the experimental frequency response estimates, and the nominal model of the system, by multiplicative uncertainty we consider:

\[
\forall P(s) \in \mathcal{P}, \quad P(s) = (1 + \Delta(s)W(s))P_0(s)
\]

(1)

Here \( W(s) \) is a fixed transfer function, called the uncertainty weighting function and \( \Delta \) is a memoryless operator of induced norm less than unity [6]. Note that in this representation \( \Delta(s)W(s) \) gives the normalized system variation away from 1 at each frequency:

\[
\frac{P(j\omega)}{P_0(j\omega)} - 1 = \Delta(j\omega)W(j\omega)
\]

(2)

Hence, since \( \|\Delta\|_\infty \leq 1 \), then

\[
\left| \frac{P(j\omega)}{P_0(j\omega)} - 1 \right| \leq |W(j\omega)|, \quad \forall \omega
\]

(3)

By plotting the system variations \( \left| \frac{P(j\omega)}{P_0(j\omega)} - 1 \right| \), for all experimental frequency response estimates of the system \( P(j\omega) \), and estimate an upper bound to those variation as a transfer function, the multiplicative uncertainty weighting function \( W(s) \), will be obtained (as illustrated in Figures 3 and 4).

As another method to obtain a linear model for the system, the nonlinear equation of motion of the system (given in [13]) can be linearized as well in a neighbourhood of origin of the state space, which can be shown to be an equilibrium point for the system. The linear model derived by this means are illustrated in Figures 3 and 4 as "Theoretical Model". The main difference between the nominal model and the theoretical model is at the resonant frequency, which is mainly due to the cancellation of the nonlinear friction terms in the linearization process of the

\footnote{Function \texttt{invfreqs} in Matlab}
theoretical model. For the purpose of control synthesis, the nominal model of the system gives better representation of the true dynamics, and thus is used for controller design. Note that by this method an effective way to find the closest linear model for a nonlinear system is proposed, and the deviation of the nonlinear system and linear model is encapsulated in model uncertainty. For harmonic drive system the uncertainty measure at low frequencies, as illustrated in Figures 3 and 4, is relatively small and about -5 db, which suggest the possibility of robustly controlling the system to perform within this bandwidth.

The nominal model for the first setup is found to be a good fit to a third order stable and minimum phase transfer function as follows:

\[
\frac{\text{Torque}}{\text{Ref Voltage}} = \frac{1.0755 \times 10^6}{s^3 + 472.7s^2 + 7.33 \times 10^4s + 5.89 \times 10^6}
\]  
(4)

which has three stable poles at \(-289.83, \text{ and } -91.44 \pm 109.48j\). The uncertainty weighting function which is the upper bound of different uncertainty frequency plots is approximated by \(W(s) = (s + 200)/356\).

For the second setup the nominal model is:

\[
\frac{\text{Torque}}{\text{Ref Voltage}} = \frac{9.673 \times 10^3}{s^2 + 142.6s + 7235.2}
\]  
(5)

which is a stable second order system with two complex conjugate poles at \(-71.29 \pm 46.39j\). The uncertainty weighting function is estimated by \(W(s) = (s + 100)/200\).

IV. ROBUST TORQUE CONTROL

Figure 5 illustrates the block diagram of the setup using multiplicative uncertainty representation, in which \(P_0\) is the nominal model of the system, \(W\) is the uncertainty weighting function, \(\Delta\) is a memoryless operator of induced \(L_2\) norm less than unity, which represents the normalized variation of the true system from the model, \(C\) is the controller. The control objective can be defined as robustly stabilizing the system, while maintaining good disturbance attenuation and small tracking error, despite the actuator saturation. More specifically, referring to Figure 5, we would like to design a controller to trade-off minimizing the norm of the transfer function from reference input \(y_d\) to the tracking error \(e\) (tracking performance), the transfer function from the disturbance \(d\) to the output \(y\) (disturbance attenuation), the transfer function from \(r\) to \(q\) (robust stability), and the transfer function from reference input \(y_d\) to the plant input \(u\) (actuator limit). This objective is well-suited to the general \(H_\infty\) problem.

Figure 6 illustrates the block diagram of the system configured for the \(H_\infty\) framework. It can be shown that tracking and disturbance attenuation objectives can be expressed as sensitivity \(S\) minimization [2]. For multiplicative uncertainty robust stability is guaranteed if the complementary sensitivity \(T\) has a norm less than unity (Small Gain Theorem [17]). \(T\) can be shown to be the transfer function from reference input \(y_d\) to the output \(y\). Weighting functions \(W_s\) and \(W_u\) are also considered to normalize and assign frequency content of the performance objectives on sensitivity and motor current saturation respectively, and \(W\) is the same as the multiplicative uncertainty weighting function. Now the augmented system has one input \(y_d\), and three outputs \(z_1, z_2, \text{ and } z_3\), in which the transfer function from the input to the outputs corresponds to weighted complementary sensitivity, weighted sensitivity, and weighted actuator effort, respectively. The objectives now will be reduced to finding the controller \(C(s)\) which minimizes the infinity norm of the transfer matrix from input \(y_d\) to the output vector \(z\) or,

\[
\text{Find } C(s) \text{ to minimize } ||T_{y_d \rightarrow z}||_{\infty}
\]  
(6)

This problem is called a mixed-sensitivity problem in the literature, and has optimal and suboptimal solution algorithms. Doyle et al. [5], provided the suboptimal solution for this problem in 1989, in which \(C(s)\) will be assigned such that \(||T_{y_d \rightarrow z}||_{\infty} < 1\). The \(\mu\)-synthesis toolbox of Matlab uses this algorithm iteratively to find the best suboptimal solution achievable [1].

Performance-weighting functions are selected considering the physical limitations of the system. The actuator saturation-weighting function is considered to be a constant, by which the maximum expected input amplitude never saturates the actuator. Its value is estimated to be 0.004 for the first setup, and 0.01 for the second setup.

The sensitivity weighting function is assigned to be \(W_s(s) = \frac{s + 100a}{2(s + a)}\), in which \(a = 3\) for first setup, and \(a = 2.8\) for second setup. This weighting function indicates that at low frequencies, the closed-loop system should reject disturbance at the output by a factor of 50 to 1. Expressed differently steady-state tracking errors due to step input should be less than 2 % or less. This performance requirement becomes less and less stringent at higher frequencies. For higher frequencies the closed-loop frequency response should degrade gracefully, always lying underneath the inverse of the weighting function \(W_s\). The best cut-off fre-
frequency for performance (a in rad/sec) is maximized by an iterative method, provided the $H_\infty$ solution to the problem exist.

Two controllers were designed using $\mu$ synthesis toolbox of Matlab. The transfer function for first setup controller is:

$$C(s) = \frac{2.08 \times 10^7(s + 289.8)(s + 91.4 \pm 109.5j)}{(s + 3)(s + 808.2 \pm 776.04j)(s + 9.8 \times 10^3)} \quad (7)$$

and for the second setup is:

$$C(s) = \frac{9.97 \times 10^6(s + 71.2943 \pm 46.3927j)}{(s + 2.8)(s + 5072.8)(s + 1.1770 \times 10^3)} \quad (8)$$

The controller zeros cancel the stable poles of the nominal plant, while the poles shape the closed-loop sensitivity function to lie beneath $W_s$.

V. CLOSED-LOOP PERFORMANCE

To verify the controller performance both simulations and experiments have been utilized. Here we only report the experimental results which are more convincing. To implement the controller in practice, bilinear discretization is performed with one kHz sampling frequency. The performance of the closed-loop system is evaluated in both frequency and time domain. The frequency domain performance of the closed-loop system is evaluated from the closed-loop frequency response of the system and is illustrated in Figure 7. For both setups the experimental sensitivity and complementary sensitivity functions are shown to be underneath the inverse of $W_s$, and $W$ respectively. Also the Nyquist plot for the loop-gain of the system is derived from the experimental sensitivity functions, and the phase margin for the first setup is found to be 60°, while for the second setup it is 52°. These results are an experimental verification of the $H_\infty$ design claim to preserve robust stability while shaping the performance as desired.

The time response of the system to different reference input signals is illustrated in Figure 8 for the first setup, and in Figure 9 for the second setup. The dotted lines are the measured output torque of the system, which is tracking the solid line, the reference command, very fast and accurately. Although our designed bandwidth was about 3 rad/sec, sinusoid inputs up to 10 Hz (62 rad/sec) are shown to be well tracked. The step response is very fast with a steady-state error less than 2% as required. Tracking of the system to triangular signal is especially sharp at the edges, and the tracking to an arbitrary signal is shown.
that the closed-loop system retains robust stability, while improving the tracking performance exceptionally well.

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**REFERENCES**


