Experimental Verification of Multi-Input Seismic Control Strategies for Smart Dampers

Fu Yi¹, Shirley J. Dyke², and Juan M. Caicedo³, and J. David Carlson ⁴

Abstract

This paper documents the results of an experimental study conducted to demonstrate the capabilities of multiple magnetorheological (MR) devices for seismic control of civil engineering structures. A six-story test structure in the Washington University Structural Control and Earthquake Engineering Lab (http://www.seas.wustl.edu/research/quake/) is considered, and four parallel-plate, shear-mode MR dampers are used to control this test structure. Two control devices are installed in the test structure between the base and first floor, and two are installed between the first floor and second floor. The system identification method used to develop a model of the integrated structural system is discussed. This method is an extension of a method successfully used in a previous semi-active experiment. Two semi-active control algorithms including a Lyapunov algorithm and a clipped-optimal algorithm are considered. An El Centro earthquake is used to disturb the system and three amplitude levels. The results indicate that high performance levels can be achieved, and the responses of the semi-active system are significantly better than that of comparable passive systems.

1. Introduction

The application of modern control techniques to mitigate the effects of seismic loads on civil engineering structures offers an appealing alternative to traditional earthquake resistant design approaches. Passive systems are widely accepted and have been successfully implemented in hundreds of structures around the world. Active and semi-active systems are being pursued because they offer the ability to modify their response to optimally reduce the responses of the structure for a wide variety of loading conditions.

Currently semi-active control devices appear to offer the best opportunity for widespread acceptance of these innovative techniques by the civil engineering community. A variety of semi-active devices have been considered for seismic applications, including variable orifice dampers, variable friction devices, adjustable tuned liquid dampers, variable stiffness dampers, and controllable fluid dampers (Spencer and Sain, 1997). Devices in this class offer the ability to dynamically vary their properties, indicating that they will be effective for a variety of loading conditions. They typically have low power requirements, eliminating the need for a large external power source. Furthermore, semi-active devices are considered to be stable because they do not have the ability to input energy into the structural system (including the control device and the structure).

1. Doctoral Candidate and Grad. Research Assistant, Department of Civil Engineering, Washington University, St. Louis, MO 63130.
2. Assistant Professor, Department of Civil Engineering, Washington University, St. Louis, MO 63130.
3. Grad. Research Assistant, Department of Civil Engineering, Washington University, St. Louis, MO 63130.
4. Engineering Fellow, Mechanical Products Division, Lord Corporation, Cary, NC 27511.
Magnetorheological (MR) dampers are classified as controllable fluid devices (Spencer et al., 1997; Carlson 1994; Carlson and Weiss, 1994; Carlson et al., 1996). These devices have demonstrated a great deal of promise for civil engineering applications in recent studies (Dyke et al., 1996c–f, 1998; Yi, et al., 1998, 1999). Both experimental and analytical studies have demonstrated that the performance of MR dampers is superior to that of comparable passive systems. Additionally, recent testing of a 20-ton MR damper at the University of Notre Dame have demonstrated that these devices can provide forces of the magnitude required for full-scale structural control applications (Spencer et al., 1997).

It is anticipated that the implementation of multiple control devices will be necessary to effectively reduce the responses of full-scale seismically-excited civil engineering structures. Furthermore, because these devices are highly nonlinear, one of the main challenges in the application of this technology is in the development of suitable control algorithms. A variety of semi-active control algorithms have been developed. Analytical investigations of a selection of these algorithms have demonstrated that the performance of a control systems based on MR dampers is highly dependent on the choice of algorithm employed (Jansen and Dyke, 1998, 1999; Dyke and Spencer 1997).

This paper presents the results of a recent experimental study that was performed to demonstrate the efficacy of multiple MR devices for seismic response control. The research focuses on an experimental structure in the Washington University Structural Control and Earthquake Engineering Laboratory. Four parallel-plate, shear-mode MR dampers are employed to control a six story test structure subjected to uniaxial ground acceleration. The procedure used to identify models of the structural system including the control devices and the structure is described herein. Verification of the model is performed by comparing the predicted results with the experimentally obtained results. Two nonlinear control algorithms are considered including a clipped-optimal controller and a Lyapunov controller. The performance of the controlled system is then examined applied to the test system, using El Centro earthquake as the excitation source. The experimental results show that the semi-active system can achieve a significant reduction in the acceleration responses at low, medium, and high excitation levels. Furthermore, comparisons of the semi-active system with a passive control configurations demonstrate that the performance of the semi-active system exceeds that of comparable passive systems.

2. Experimental Setup

Experimental investigations were performed in the Washington University Structural Control and Earthquake Engineering Laboratory. This experimental facility houses a uniaxial seismic simulator and was established to provide a testbed for experimental verification of innovative seismic control techniques. The simulator consists of a 1.7×1.7 m² (5×5 ft²) aluminum sliding table mounted on high-precision, low-friction, linear bearings.

The MR devices employed in this experiment are prototype devices, shown schematically in Fig. 1. The experimental device were obtained from the Lord Corporation for testing and evaluation (http://www.mrfluid.com/). The device consists of two steel parallel plates. The dimensions of the device are 4.45×1.9×2.5 cm³ (1.75×0.75×1.0 in³). The magnetic field produced in the device is generated by an electromagnet consisting of a coil at one end of the device. Forces are generated when the moving plate, coated with a thin foam saturated with MR fluid, slides between the two parallel plates.
The outer plates of the MR device are 0.635 cm (0.25 in) apart, and the force capacity of the device is dependent on the strength of the fluid and on the size of the gap between the side plates and the center plate. In this experiment, a center plate with a thickness of 0.495 cm (0.195 in) is selected resulting in a gap of 0.071 cm (0.028 in). Each of the control devices can generate a maximum force of 29 N, which is approximately 1.6% the weight of the structure.

Power is supplied to the device by a regulated voltage power supply driving a DC to pulse-width modulator (PWM). The PWM supplies voltage pulses to the MR damper at a frequency greater than 20 kHz, and the command voltage to the circuit controls the duty cycle of the individual pulses. The circuit has been calibrated such that a 4V command signal corresponds to 100% duty cycle and a 0V command signal corresponds to a 0% duty cycle.

The test structure used in this experiment is a six-story, single bay, steel frame (Fig. 2). The structure is 180 cm (74 in) tall and has a mass of 147 kg (325 lb) which is distributed uniformly among the floors. Parallel-plate MR dampers are placed between the ground and first floor, and between the first and second floors of the structure. Two MR dampers of this type are used in each location.

Sensors are installed in the model building for use in determining the control action. PCB Piezotronics capacitive accelerometers on all six floors of the structure provide measurements of the absolute accelerations, and a force transducer placed in series with one MR damper on each floor measures the control force \( f \) being applied to the structure. Note that only these eight measurements are used in the control algorithms. Additionally, two LVDT’s are located in the structure to measure the displacements of the lower two floors and the MR dampers. A schematic diagram of the experimental setup is shown in Fig. 3.

A 16-channel data acquisition system made by DSP Technology is used for data acquisition and system evaluation. Control algorithms are implemented on a DSP-based real-time control system by dSpace, Inc.


3. SYSTEM IDENTIFICATION

The first step in designing a control system is the identification of a model of the system to be controlled. In this experiment, the system to be controlled consists of the six story test structure and the MR dampers. A modification of the approach used by Spencer and Dyke (1996) was used to identify a model of the system. In this method one first considers each of the components of the system, and then develops an integrated model of the system. The steps in this process include, i) identifying a model of the MR device, ii) identifying a model of the test structure, iii) developing

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![Figure 2. Photograph of Test Structure.](image)

![Figure 3. Schematic Diagram of Test Setup.](image)
an integrated model of the system, and iv) verification of the integrated model. Each of these steps are described in detail in the following sections.

3.1 Modeling and Identification of the Shear Mode MR Damper

The first step in developing a model of the MR device was to obtain experimental responses of the MR damper. A load frame was constructed to obtain this data. The load frame employs a 2 kip Shore Western hydraulic actuator to apply forces to the MR device. The damper is placed in series with a force transducer made by PCB Piezotronics. A variety of displacement and voltage combinations were input to the MR damper. The maximum force generated in the device was approximately 29 N. The dynamic range (defined as the ratio of the peak force with a maximum control input of 4 V to the peak force with a 0 V input) observed in the device used in this experiment was approximately 7. The MR effect begins to saturate at approximately 3.5 V. The rise time of the system, primarily determined by the resistance and inductance of the electromagnet itself, was measured to be approximately 50 msec.

Spencer et al., (1997) recently developed a phenomenological model of an MR device. The damper used in that study used MR fluid in valve-mode, and consisted of a piston device. The MR devices employed in this study are parallel-plate, shear-mode dampers. These devices have similar, but not the same, characteristics as the devices studied in those previous experiments. Thus, modifications were made to obtain a phenomenological model of this device. The resulting mechanical model is shown in Fig. 5. The force in this system is given by

\[
\begin{align*}
\text{Figure 4. Typical Responses of the Shear Mode MR Damper} \\
\text{Figure 5. Mechanical Model of the Parallel-Plate MR Damper.}
\end{align*}
\]
\[ f = c_0 \dot{x} + \alpha z \]  

where \( x \) is the displacement of the damper, and the evolutionary variable \( z \) is governed by

\[ \dot{z} = -\gamma |\dot{x}|z|z|^{n-1} - \beta |\dot{x}|z^n + A \dot{x}. \]  

By adjusting the parameters of the model \( \gamma, \beta, n, \) and \( A \), one can control the linearity in the unloading and the smoothness of the transition from the pre-yield to the post-yield region.

To use the device for control purposes, a model is required that is capable of predicting the behavior of the MR damper for a time-varying command input. Thus, the functional dependence of the parameters on the command voltage was determined. The following relations are proposed

\[ \alpha = \alpha(u) = \alpha_a + \alpha_b u \quad \text{and} \quad c_0 = c_0(u) = c_{0a} + c_{0b} u. \]  

In addition, as mentioned previously, the resistance and inductance in the circuit introduces dynamics into the system. These dynamics are observed as a first order time lag in the response of the device to changes in the command input. These dynamics are accounted for through the first order filter

\[ \dot{u} = -\eta(u - v) \]  

where \( v \) is the command voltage applied to the PWM circuit.

A constrained nonlinear optimization was used to obtain the ten parameters. The optimization was performed using a sequential quadratic programming algorithm available in MATLAB (1997). The initial set of optimized parameters that were determined to best fit the experimental data in a variety of representative tests are: \( \alpha_a = 27.3 \text{ N/cm}, \quad \alpha_b = 26.5 \text{ N/cm} \cdot \text{V}, \quad c_{0a} = 0.032 \text{ N} \cdot \text{sec/cm}, \quad c_{0b} = 0.02 \text{ N} \cdot \text{sec/cmV}, \quad n = 1, \quad A = 120, \quad \gamma = 300 \text{ cm}^{-2}, \quad \beta = 300 \text{ cm}^{-1}, \quad \text{and} \quad \eta = 80 \text{ sec}^{-1}.

### 3.2 Identification of the Test Structure

To achieve the objective of developing a model of the integrated structural system (consisting of the structure and the control devices), a model of the structure alone is first developed. The structure to be identified is a MIMO (multi-input, multi-output) system. The three inputs to the structure include the ground acceleration, the control force applied at the first floor, and the control force applied between the first and second floors. This linear system can be described by the \([6 \times 3]\) transfer function matrix
where $s$ is the Laplace variable. Each column of this matrix corresponds to an input to the structure and each row corresponds to a measured output.

A variation of the method developed by Dyke et al., (1996,a,b,d) was used to identify a model of the test structure. In this approach, an external device is used to independently excite the structure at locations corresponding to each control input. Because the system to be identified in this experiment is a MIMO system, an actuator would be needed to independently excite the system at the location of each control input. Using an actuator to excite the structure between the first and second floors of the structure would add mass to the structure, altering the transfer functions of the system, and resulting in an inadequate model of the system. Thus, this method cannot be directly applied to identify the structure in this experiment, and a modified system identification procedure was developed for this experiment.

The steps in the modified approach are: a) experimental determination of transfer functions from the ground acceleration input to each of the outputs, b) mathematical modeling of each of the transfer functions as a ratio of polynomials in the Laplace variable $s$, c) development of a MIMO state space realization of the system based on a lumped mass model of the structure, and d) experimental verification of the structural model.

The transfer functions from the ground acceleration to each of the measured structural accelerations, the first column of the transfer function matrix (Eq. 5), were experimentally determined by exciting the structure with the seismic simulator. Each of these six transfer functions were then modeled as a ratio of polynomials in the Laplace variable $s$ using the program described in (Dyke et al., 1996a,b). Twelve poles were used to model each of the transfer functions. A state space model of the SIMO (single-input, multi-output) system from the ground acceleration to the measured structural responses was formed. The first six natural frequencies of the structure are 1.28 Hz, 3.83 Hz, 6.19 Hz, 8.25 Hz, 9.79 Hz, and 10.96 Hz. Representative comparisons of the transfer functions of the identified system and the experimentally obtained transfer functions are shown in the Fig. 6.

Because the experimental transfer functions from the control force inputs to the structural responses (the second and third columns of the transfer function matrix) were not available, a lumped mass model of the six-story test structure was developed and used to determine a model of the system from the control inputs. Comparing the transfer functions obtained experimentally to those of the lumped mass model, we found that the transfer functions were quite similar, and the all of the poles of the experimental structure were within 8% of those found in the lumped mass model. The eigenvalues of the lumped mass model were modified using the system transformation matrix. To verify the model, a comparison was then performed in the time domain using experimental data and predicted structural responses to a measured ground acceleration.
3.3 Development of an Integrated System Model

The next step is to optimize the set of parameters for the MR damper models for the case when it is installed in the test structure, and combine the models of the device and structure to form the integrated system model. It is necessary to update the parameters of the MR damper model because when the MR damper is employed in the test structure it may be functioning at a different operating point than in the initial tests in which the damper was driven with a hydraulic actuator.

To identify a new set of parameters, a series of tests was conducted to measure the response of the system with the MR dampers in the test structure. In these tests, the base of the structure was excited and a variety of command signals were applied to the MR damper similar to those used in the load frame tests. Three cases were tested including: 1) two MR dampers installed between the base and first floor, 2) two MR dampers installed between the first and second floors, and 3) two MR dampers on each of the first two floors of the structure. The recorded system responses include the force generated in each MR damper, absolute accelerations of the six floors of the structure, displacements of each control device, and acceleration of the base. A least-squares output-error method was employed in conjunction with a constrained nonlinear optimization to obtain the updated model parameters required to model the MR damper. The resulting optimized parameters were the same as those found in the load frame tests except for $\alpha_{0a}$ and $\alpha_{0b}$. These parameters varied between the four dampers, and were in the ranges $\alpha_{0a} \in [12.32, 15.95]$ N/cm and $\alpha_{0b} \in [15.59, 20.91]$ N/(cm·V).

![Figure 6. Comparison of the Analytical (--) and Experimental (--) Transfer Functions from the Ground Acceleration to the First Floor Absolute Acceleration.](image-url)

$\alpha_{0a} = 12.32, 15.95$, $\alpha_{0b} = 15.59, 20.91$.
3.4 Verification of the Integrated System Model

To verify that the identified model is adequate for control synthesis and analysis, the predicted results and experimental response were compared in several cases. A representative comparison between the identified nonlinear system model and the experimental data for an El Centro earthquake input and a time-varying command input is shown in Fig. 7.

![Figure 7. Experimental and Predicted Sixth Floor Absolute Acceleration with El Centro Ground Acceleration in Controlled Test (Clipped-Optimal).](image)

4. CONTROL ALGORITHMS

Semi-active control systems are highly nonlinear and require appropriate nonlinear control algorithms that can take advantage of key features of the control devices. Additionally, to be implementable in full-scale applications, an algorithm should use available measurements in determining the control action. Based on the results of a recent numerical study to evaluate the performance of a variety of semi-active control algorithms (Jansen and Dyke, 1999a,b) two versatile and effective control algorithms were selected for consideration in the experiment. These two control algorithms, the clipped-optimal controller (Dyke et al., 1996) and the Lyapunov controller (Leitmann, 1994), are described in this section.

Consider a seismically excited structure controlled with $n$ MR dampers. Assuming that the forces provided by the control devices are adequate to keep the response of the primary structure from exiting the linear region, the equations of motion can be written in the following state-space form

$$\dot{z} = Az + Bf + E\ddot{g}$$  \hspace{1cm} (6)

$$y = Cz + Df + v$$  \hspace{1cm} (7)
where $\ddot{x}_g$ is a one-dimensional ground acceleration, $\mathbf{f} = [f_1, f_2, \ldots, f_n]'$ is the vector of measured control forces, $\mathbf{z}$ is the state vector, $\mathbf{y}$ is the vector of measured outputs, and $\mathbf{v}$ is the measurement noise vector. For these applications, the measurements typically available for control force determination include the absolute acceleration of selected points on the structure, the displacement of each control device, and a measurement of each control force.

In developing the control laws, the following assumptions were used: i) the control voltage to the $i$th device is restricted to the range $v_i = [0, V_{max}]$, and ii) for a fixed set of states, the magnitude of the applied force $|f_i|$ increases when $v_i$ increases, and decreases when $v_i$ decreases.

4.1 Control Based on Lyapunov Stability Theory

In some cases it is possible to employ Lyapunov’s direct approach to stability analysis in the design of a feedback controller (Brogan, 1991). The approach requires the use of a Lyapunov function, denoted $V(z)$, which must be a positive definite function of the states of the system, $\mathbf{z}$. According to Lyapunov stability theory, if the rate of change of the Lyapunov function, $\dot{V}(\mathbf{z})$, is negative semi-definite, the origin is stable i.s.L. (in the sense of Lyapunov). Thus, in developing the control law, the goal is to choose control inputs for each device that will result in making $\dot{V}$ as negative as possible. An infinite number of Lyapunov functions may be selected, that may result in a variety of control laws.

Leitmann (1994) applied Lyapunov’s direct approach for the design of a semi-active controller. In this approach, a Lyapunov function is chosen of the form

$$V(\mathbf{z}) = \frac{1}{2}||\mathbf{z}||_P^2$$

where $||\mathbf{z}||_P$ is the $P$-norm of the states defined by

$$||\mathbf{z}||_P = [\mathbf{z}'P\mathbf{z}]^{1/2}$$

and $\mathbf{P}$ is a real, symmetric, positive definite matrix. In the case of a linear system, to ensure $\dot{V}$ is negative definite, the matrix $\mathbf{P}$ is found using the Lyapunov equation

$$A'\mathbf{P} + \mathbf{PA} = -\mathbf{Q}_P$$

for a positive definite matrix $\mathbf{Q}_P$. The derivative of the Lyapunov function for a solution of Eq. (6) is
\[
\dot{V} = -\frac{1}{2} z' Q_p z + z' P B f_i + z' P E x_g
\]  

(11)

Thus, the control law which will minimize \( V \) is

\[
v_i = V_{\text{max}} H(-z' P B_i f_i)
\]

(12)

where \( H(\cdot) \) is the Heaviside step function, \( f_i \) is the measured force produced by the \( i \)th MR damper, and \( B_i \) is the \( i \)th column of the \( B \) matrix in Eq. 1. Notice that this algorithm falls into the class of bang-bang controllers. The command signal is dependent only on the sign of the measured control force and the states of the system.

To implement this algorithm, a Kalman filter is used to estimate the states based on the available measurements (i.e., device forces, structural accelerations). Thus, in this algorithm, higher performance levels are expected when measurements of the responses of the full structure are used. However, one challenge in the use of the Lyapunov algorithm is in the selection of an appropriate \( Q_p \) matrix. In this study a variety of \( Q_p \) matrices were tested in simulation to identify an effective design.

4.2 Clipped-Optimal Control Design

Dyke, et al. (1996c–e, 1998) have proposed a clipped-optimal control strategy based on acceleration feedback for the MR damper. Analytical and experimental studies have demonstrated that the MR damper, used in conjunction with the clipped-optimal control algorithm, was effective for controlling a multi-story structure with a single MR damper (Dyke et al., 1996c, 1998). Dyke and Spencer (1996) then extended the control algorithm to control multiple MR devices, and performed a numerical study demonstrating the efficacy of the control algorithm. This extension of the control algorithm is described herein.

In the clipped-optimal controller, the approach is to append \( n \) force feedback loops to induce each MR damper to generate approximately a specified control force. The desired control force of the \( i \)th MR damper is denoted \( f_{ci} \). A linear optimal controller \( K_c(s) \) is designed that calculates a vector of desired control forces, \( f_c = [f_{c1}, f_{c2}, \ldots, f_{cn}]' \), based on the measured structural response vector \( y \) and the measured control force vector \( f \), i.e.,

\[
\begin{align*}
K_c(s) = & L^{-1} \
& \left\{ -K_c(s) L \begin{bmatrix} y \\ f \end{bmatrix} \right\}
\end{align*}
\]

(13)

where \( L \{ \cdot \} \) is the Laplace transform. Although the controller \( K_c(s) \) can be obtained from a variety of synthesis methods, \( H_2 \)/LQG strategies are advocated herein because of the stochastic na-
ture of earthquake ground motions and because of their successful application in other civil engineering structural control applications (Dyke et al., 1996a–f).

To discuss the algorithm used for determining the control action, consider the $i$th MR damper used to control the structure. Because the response of the MR damper is dependent on the relative structural displacements and velocities at the point of attachment of the MR damper, the force generated by the MR damper cannot be commanded, only the voltage $v_i$ applied to the current driver of the $i$th MR damper can be directly controlled. To induce the MR damper to generate approximately the corresponding desired optimal control force $f_{ci}$, the command signal $v_i$ is selected according to the control law

$$v_i = V_{\text{max}} H(f_{ci} - f_i)$$

where $V_{\text{max}}$ is the maximum voltage to the PWM circuit, and $H(\cdot)$ is the Heaviside step function.

One of the attractive features of this control strategy is that the feedback for the controller is based on readily obtainable acceleration measurements, thus making it quite implementable. In addition, the proposed control design does not require a model for the MR damper, although the model of the damper is important to system analysis.

5. EXPERIMENTAL RESULTS

To demonstrate the effectiveness of these multi-input control strategies, experiments were conducted on a six-story test structure in Washington University Structural Control and Earthquake Lab. The six story model structure was subjected to scaled versions of the 1940 El Centro earthquake and structural acceleration responses were recorded. The tests were performed at three different excitation amplitudes because the MR damper is nonlinear device and will achieve different performance levels at different amplitudes. The maximum amplitude was selected as the largest amplitude that will not damage the structure. Levels of 55%, 105% and 125% of the recorded El Centro earthquake were selected in this experiment. These will be referred to as low, medium, and high amplitude excitation tests, respectively.

Both a Lyapunov and a clipped-optimal controller were used to control the structure. The Lyapunov controller was designed by placing ones in the (7,1), (8,2), (9,3), (10,4), (11,5), and (12,6) positions of the $Q_p$ matrix. The clipped optimal controller was designed by placing a weighting of 4600 on the top floor absolute acceleration of the structure.

To demonstrate the potential advantages of semi-active control systems over comparable passive systems, the results of two passive cases are included. Passive-off and passive-on refer to the cases in which the voltage to the MR damper is held at a constant value of $V = 0$ V and $V = V_{\text{max}} = 4.0$ V, respectively. Note that the control forces in the passive-off case are relatively small and have little effect on the responses of the structure. Thus, in the remainder of this article, uncontrolled will refer to the passive-off case.

The various control algorithms were evaluated using the evaluation criteria from the second generation control problem for buildings (Spencer et al., 1999). A subset of the evaluation criteria were selected based on the structural responses that were available for measurement in this exper-
iment. The first evaluation criterion considers the normalized peak floor absolute accelerations, given as

\[
J_1 = \max_{i,t} \left( \frac{\ddot{x}_{ai}(t)}{\ddot{x}_{a}^{\max}} \right)
\]

(15)

where the absolute acceleration of the \(i\)th floor of the structure, \(\ddot{x}_{ai}(t)\), is normalized by the peak uncontrolled floor acceleration, denoted \(\ddot{x}_{a}^{\max}\).

The second evaluation criterion is a measure of the normed floor absolute acceleration response, given as

\[
J_2 = \max_{i,t} \left( \frac{\|\ddot{x}_{ai}(t)\|}{\|\ddot{x}_{a}^{\max}\|} \right)
\]

(16)

where \(\|\ddot{x}_{ai}(t)\| = \int_0^T \ddot{x}_{ai}(t) \, dt\), and the absolute accelerations of the \(i\)th floor, \(\ddot{x}_{ai}(t)\), are normalized by the peak uncontrolled floor acceleration, denoted \(\ddot{x}_{a}^{\max}\).

The third evaluation criterion considers the maximum base shear generated in the controlled configuration and is given as

\[
J_3 = \max_t \left| \sum_{i=1}^{6} m_i \ddot{x}_{ai}(t) \right| / F_{b}^{\max}
\]

(17)

\(F_{b}^{\max}\) is the maximum base shear in the uncontrolled configuration. In this experiment the mass of the \(i\)th floor, \(m_i\), is a constant 24.52 kg (54 lb).

The normed/non-dimensionalized base shear is used as the fourth evaluation criterion such that

\[
J_4 = \left\| \sum_{i=1}^{6} m_i \ddot{x}_{ai}(t) \right\| / \left\| F_{b}^{\max} \right\|
\]

(18)

\(\left\| F_{b}^{\max} \right\| = \sum_{i=1}^{6} m_i \ddot{x}_{ai}(t)\) is the maximum normed uncontrolled base shear.

The last evaluation criteria considered in this study is a measure of the maximum control force per device, given by
where $f_i(t)$ is the force generated by the $i$th control device over the time history of each earthquake, and $W = 1446$ N (325 lbf) is the weight of the structure.

Table 1 provides the peak and normed experimental results of the uncontrolled structure. These values are used to calculate the evaluation criteria in Eqs. (15–19).

<table>
<thead>
<tr>
<th>Amplitude</th>
<th>$x_{a\text{max}}$ (cm/sec$^2$)</th>
<th>$|x_{a\text{max}}|$ (cm/sec$^2$)</th>
<th>$F_{b\text{max}}$ (N)</th>
<th>$|F_{b\text{max}}|$ (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>High</td>
<td>371.8</td>
<td>4803</td>
<td>150.38</td>
<td>2438.30</td>
</tr>
<tr>
<td>Medium</td>
<td>231.6</td>
<td>2660</td>
<td>97.17</td>
<td>1411.16</td>
</tr>
<tr>
<td>Low</td>
<td>100.7</td>
<td>1029</td>
<td>40.92</td>
<td>568.38</td>
</tr>
</tbody>
</table>

The experimental results at the three excitation levels in the controlled configuration are provided in Table 2. At both the high and medium excitation levels, the passive-on system reduces the structural responses as compared to the uncontrolled results. However, at low excitation amplitudes, the passive-on system results in a significant increase in the structural accelerations. The peak acceleration is increased by 70% and the normed acceleration is increased by 40%. Furthermore, the peak base shear is increased slightly, while the normed base shear is increased by 20%. Thus, the results demonstrate that simply applying the maximum voltage to generate the largest control forces is not typically the most effective scheme for reducing the seismic responses of the structure.

The Lyapunov controller achieves higher performance levels than that of the passive-on system at all amplitude levels. At high amplitude levels a 30% reduction in the peak acceleration and a 50% reduction in the normed acceleration was observed as compared to the uncontrolled results. This corresponds to a 21% and a 23% reduction over the passive-on results, respectively. In this case, the Lyapunov controller also achieves a 45% and 49% reduction in the peak and normed base shear values, respectively. At medium amplitude levels the Lyapunov controller achieves a

\begin{equation}
J_S = \max_{t, i} \left( \frac{|f_i(t)|}{W} \right)
\end{equation}
19% reduction in peak acceleration and 36% reduction in normed acceleration as compared to the uncontrolled case, and a 20% reduction in the peak acceleration and a 20% reduction in the normed acceleration as compared to the passive-on case. The peak and normed base shear criteria are reduced by 42% and 40%, respectively. This result corresponds to a 22% and 21% reduction in these responses, respectively, above that of the passive-on case. In the low amplitude case the peak accelerations are only slightly smaller than the passive-off case, although the peak and normed base shear criteria are reduced by 27% and 23%, respectively. In the low amplitude tests all response criteria were increased in the passive-on case.

The clipped-optimal controller achieves similar performance levels. At high amplitude levels a 12% reduction in the peak acceleration and a 45% reduction in the normed acceleration was observed as compared to the uncontrolled results. This corresponds to a 44% reduction over the normed acceleration in the passive-on case. At medium amplitude levels the Lyapunov controller achieves a 24% reduction in peak acceleration and 37% reduction in normed acceleration as compared to the uncontrolled case, and a 40% reduction in the peak acceleration and a 22% reduction in the normed acceleration as compared to the passive-on case. Furthermore, the peak and normed base shear criteria are reduced by 21% and 19%, respectively, beyond that of the passive-on case. In the low amplitude case the peak accelerations are reduced by 15% and the normed accelerations are reduced by 24% over the passive-off case. Additionally, the peak and normed base shear

<table>
<thead>
<tr>
<th>Control Strategy</th>
<th>$J_1$</th>
<th>$J_2$</th>
<th>$J_3$</th>
<th>$J_4$</th>
<th>$J_5$</th>
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<tr>
<td><strong>High Amplitude El Centro Earthquake (125%)</strong></td>
<td></td>
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<tr>
<td>Passive-Off</td>
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<td>1.0000</td>
<td>1.0000</td>
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<td>0.0045</td>
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<td>Passive-On</td>
<td>0.8978</td>
<td>0.6276</td>
<td>0.6980</td>
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<tr>
<td>Clipped-Optimal</td>
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<td>0.5471</td>
<td>0.6573</td>
<td>0.6049</td>
<td>0.0165</td>
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<tr>
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<td>0.4996</td>
<td>0.5453</td>
<td>0.5144</td>
<td>0.0154</td>
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<tr>
<td><strong>Medium Amplitude El Centro Earthquake (105%)</strong></td>
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<td>0.0042</td>
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<tr>
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<tr>
<td><strong>Low Amplitude El Centro Earthquake (55%)</strong></td>
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<tr>
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<td>0.7982</td>
<td>0.8337</td>
<td>0.8661</td>
<td>0.0125</td>
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</table>
responses are 37% and 28% lower, respectively, than the best passive case. However the peak and normed accelerations are 50% and 46% smaller than the passive-on results, respectively.

In comparing the performance of the Lyapunov and clipped-optimal controllers, observe that the normed responses are very similar at all amplitude levels. However, the peak responses were somewhat different. At both low and medium amplitudes the clipped-optimal controller outperforms the Lyapunov controller in reducing the peak acceleration. At high excitation levels the Lyapunov controller did not achieve the performance of the clipped-optimal controller.

Response profiles of the peak acceleration measurements are shown in Figure 8. These plots clearly demonstrate the superior behavior of the semi-active system. Not only are the maximum controlled responses lower than the passive cases, but typically the peak responses of the entire structure are reduced significantly. For high and medium excitation levels, the semi-active control systems results in smaller peak accelerations than both the passive-on and passive-off cases. A representative time history comparing the passive-on and the clipped-optimal responses is shown in Fig. 9. Furthermore, the passive-on system results in an increase in the peak accelerations at all floors of the structure over the passive-off case, while the clipped-optimal and Lyapunov controllers achieve significant reduction in the peak accelerations throughout the structure. This result is further demonstrated in Figures 10 and 11, which compare the third and sixth floor acceleration responses, respectively, for the passive-on and clipped-optimal cases at the small excitation level. Notice that the clipped-optimal controller not only reduces the peak response, but achieves a significant reduction in the accelerations throughout the earthquake.

Interestingly, notice that for all of the semi-actively controlled systems, these performance gains are achieved while requiring smaller control forces than are required in the passive-on case.

![Figure 8. Acceleration Peak and Normed Acceleration Response Profiles.](image)
6. CONCLUSION

This paper documents the results of an experimental study conducted to demonstrate the efficacy of two semi-active control algorithms on a system using multiple semi-active control devices. The semi-active control device used herein is the magnetorheological damper. The experiments were conducted on a six-story test structure in the Washington University Structural Control and Earthquake Engineering Laboratory. The control system employs four MR dampers to control the six story structure. Each of the control devices is capable of supplying a maximum force of 29 N, which is only 1.6% the weight of the entire structure.
A four step system identification procedure was used to identify an analytical model of the structural system. This method is an extension of a method found to be successful in previous active and semi-active experiments. Two semi-active control algorithms were considered, including a clipped-optimal algorithm and a Lyapunov algorithm. Tests conducted at various excitation levels demonstrated the ability of the MR damper to surpass the performance of a comparable passive system in a variety of situations. Furthermore, using only a very small amount of power, the semi-active systems is superior to comparable passive systems.

Experimental results, and video clips that visually document the results of this experiment are available at http://www.seas.wustl.edu/research/quake/.

7. ACKNOWLEDGMENTS

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8. REFERENCES


