Introduction to System Identification and Adaptive Control

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• Introduction to Adaptive Control

  ▪ Control System Design Aims to Achieve:

    1- Closed Loop Stability
    2- Desired Closed Loop Performance (Both Transient and Steady State)
• **Some Facts**

- Real Industrial Plants are Complex in Nature
- Perfect Modeling Not Feasible
- Variations of System Parameters with Time
- Model Structure Deficiency: Uncertainty
- Disturbances and Unknown Noises

• **The Feedback Problem**

Control systems are designed to maintain closed loop stability with desired closed loop performance in the presence of:

  - Model Uncertainty
  - Time Varying Parameters
  - Disturbances & Unknown Noises
• **The Control Engineer Solution Packages:**

- **Robust Control** ➔ **LTI Structure**
  Limited Performance, Strong Mathematical Foundation
- **Adaptive Control** ➔ **NLTV Structure**
  Nearly Unlimited Performance
  Mathematical Foundation
- **Intelligent Control** ➔ **NLTV Structure**
  Soft computing Mathematical Foundation

• **Definition:**

**To Adapt**
- Behavioral Change in order to adjust to new conditions

**Adaptive Controller**
- A controller capable of readjusting its functioning for response to changes in system dynamics or disturbance input
• **A Short historical perspective**

✓ Start in 1950’s: Auto Pilot design for Flight Control

✓ Fast Dynamical Changes
✓ High Performance Behaviour
✓ Trial and Error Methods Without Concrete Theoretical basis
✓ Plane Crash Accident
✓ First Symposium Till 1981
✓ Kalman Self-tuning Controller(1958)
✓ Honeywell + General Electric

• **Two decades of Background Preparations**

✓ 1960’s: Theoretical Basis for Stability Assessment of Adaptive Systems

✓ Lyapunov Stability Analysis
✓ State Space Analysis
✓ Stochastic Control
✓ Discrete Time Systems
✓ System Identification: Research Commencement and Basic Understanding
1970’s: Stability Analysis and Convergence of Adaptive Systems

Lyapunov Stability Theorem
I/P-O/P Stability

and

Stable Adaptive Control

Proof of Convergence Theorems
Under Solid Conditions

1980’s: Robust Adaptive Control

From 1990’s:

- More Accurate Proofs for Stability, Convergence and Robustness Theorems
- Artificial Intelligence, Neural Networks, and Fuzzy logic
- Combined Methods
Effect of Parameter Change in Systems

**EXAMPLE 1.1** Different open-loop responses

Consider systems with the open-loop transfer functions

\[ G(s) = \frac{1}{(s + a)(s + b)} \]

where \(a = -0.01, 0, \text{and} \ 0.01\). The dynamics of these processes are quite different, as is illustrated in Fig. 1.4(a). Notice that the responses are significantly different. The system with \(a = 0.01\) is stable; the others are unstable. The initial parts of the step responses, however, are very similar for all systems. The closed-loop systems obtained by introducing the proportional feedback with unit gain, that is, \(u = u - y\), give the step responses shown in Fig. 1.4(b). Notice that the responses of the closed-loop systems are virtually identical. Some insight is obtained from the frequency responses. Bode diagrams for the

![Figure 1.4](image)

Figure 1.4 (a) Open-loop unit step responses for the process in Example 1.1 with \(a = -0.01, 0, \text{and} \ 0.01\). (b) Closed-loop step responses for the same system, with the feedback \(u = u - y\). Notice the difference in time scales.

![Figure 1.5](image)

Figure 1.5 (a) Open-loop and (b) closed-loop Bode diagrams for the process in Example 1.1.
EXAMPLE 1.2  Similar open-loop responses
Consider systems with the open-loop transfer functions

\[ G(s) = \frac{400(1 - T)}{(s + 1)(s + 20)(1 + Ts)} \]

with \( T = 0, 0.015, \) and 0.03. The open-loop step responses are shown in Fig. 1.6(a). Figure 1.6(b) shows the step responses for the closed-loop systems obtained with the feedback \( u = u_r - y.\) Notice that the open-loop responses

![Diagram of open-loop responses](image1)

**Figure 1.6** (a) Open-loop unit step responses for the process in Example 1.2 with \( T = 0, 0.015, \) and 0.03. (b) Closed-loop step responses for the same system, with the feedback \( u = u_r - y.\) Notice the difference in time scales.

![Diagram of Bode plots](image2)

**Figure 1.7** Bode diagrams for the process in Example 1.2. (a) The open-loop system. (b) The closed-loop system.
Nonlinear Actuators

A very common source of variations is that actuators, like valves, have a nonlinear characteristic. This may create difficulties, which are illustrated by the following example.

EXAMPLE 1.4 Nonlinear valve

A simple feedback loop with a Proportional and Integrating (PI) controller, a nonlinear valve, and a process is shown in Fig. 1.8. Let the static valve characteristic be

\[ v = f(u) = u^4 \quad u \geq 0 \]

Linearizing the system around a steady-state operating point shows that the incremental gain of the valve is \( f'(u) \), and hence the loop gain is proportional to \( f'(u) \). The system can perform well at one operating level and poorly at another. This is illustrated by the step responses in Fig. 1.9. The controller is tuned to give a good response at low values of the operating level. For higher values of the operating level the closed-loop system even becomes unstable.

One way to handle this type of problem is to feed the control signal \( u \) through an inverse of the nonlinearity of the valve. It is often sufficient to use a fairly crude approximation (see Example 9.1). This can be interpreted as a special case of gain scheduling, which is treated in detail in Chapter 9.

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**Figure 1.8** Block diagram of a flow control loop with a PI controller and a nonlinear valve.

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**Figure 1.9** Step responses for PI control of the simple flow loop in Example 1.4 at different operating levels. The parameters of the PI controller are \( K = 0.15, T_i = 1 \). The process characteristics are \( f(u) = u^4 \) and \( G_p(s) = 1/(s + 1)^4 \).
Flow and Speed Variations

Systems with flows through pipes and tanks are common in process control. The flows are often closely related to the production rate. Process dynamics thus change when the production rate changes, and a controller that is well tuned for one production rate will not necessarily work well for other rates. A simple example illustrates what may happen.

**EXAMPLE 1.5 Concentration control**

Consider concentration control for a fluid that flows through a pipe, with no mixing, and through a tank, with perfect mixing. A schematic diagram of the process is shown in Fig. 1.10. The concentration at the inlet of the pipe is $c_i$. Let the pipe volume be $V_p$ and let the tank volume be $V_t$. Furthermore, let the flow be $q$ and let the concentration in the tank and at the outlet be $c$. A mass balance gives

$$V_p \frac{dc(t)}{dt} = q(t) (c_i (t - \tau) - c(t)) \quad (1.3)$$

where

$$\tau = V_p / q(t)$$

![Figure 1.10 Schematic diagram of a concentration control system.](image)

Introduce

$$T = V_p / q(t) \quad (1.4)$$

For a fixed flow, that is, when $q(t)$ is constant, the process has the transfer function

$$G_p(s) = \frac{e^{-\tau s}}{1 + sT} \quad (1.5)$$

The dynamics are characterized by a time delay and first-order dynamics. The time constant $T$ and the time delay $\tau$ are inversely proportional to the flow $q$.

The closed-loop system is as in Fig. 1.8 with $f(t) = 1$ and $G_v(s)$ given by Eq. (1.5). A controller will first be designed for the nominal case, which corresponds to $q = 1$, $T = 1$, and $\tau = 1$. A PI controller with gain $K = 0.5$ and integration time $T_i = 1.1$ gives a closed-loop system with good performance in this case. Figure 1.11 shows the step responses of the closed-loop system for different flows and the corresponding control actions. The overshoot will increase with decreasing flows, and the system will become sluggish when the flow increases. For safe operation it is thus good practice to tune the controller at the lowest flow. Figure 1.11 shows that the system can easily cope with a flow change of ±10% but that the performance deteriorates severely when the flow changes by a factor of 2.

Variations in speed give rise to similar problems. This happens for example in rolling mills and paper machines.
EXAMPLE 1.6 Short-period aircraft dynamics

A schematic diagram of an airplane is given in Fig. 1.12. To illustrate the effect of parameter variations, we consider the pitching motion of the aircraft. Introduce the pitch angle $\theta$. Choose normal acceleration $N_x$, pitch rate $q = \dot{\theta}$, and elevator angle $\delta$, as state variables and the input to the elevator serve as the input signal $u$. The following model is obtained if we assume that the aircraft is a rigid body:

$$\frac{dx}{dt} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & -\sigma \end{pmatrix} x + \begin{pmatrix} b_1 \\ 0 \\ \sigma \end{pmatrix} u \quad (1.6)$$

where $x^T = \begin{pmatrix} N_x & \theta & \delta \end{pmatrix}$. This model is called short-period dynamics. The parameters of the model given depend on the operating conditions, which can be described in terms of Mach number and altitude; see Fig. 1.13, which shows the flight envelope.
the flight envelope.

Table 1.1 shows the parameters for the four flight conditions (FC) indicated in Fig. 1.13. The data applies to the supersonic aircraft F4-E. The system has three eigenvalues. One eigenvalue, $\lambda = -14$, which is due to the elevon servo, is constant. The other eigenvalues, $\lambda_1$ and $\lambda_2$, depend on the flight conditions. Table 1.1 shows that the system is unstable for subsonic speeds (FC 1, 2, and 3) and stable but poorly damped for the supersonic condition FC 4. Because of these variations it is not possible to use a controller with the same parameters for all flight conditions. The operating condition is determined from air data sensors that measure altitude and Mach number. The controller parameters are then changed as a function of these parameters. How this is done is discussed in Chapter 9.

Much more complicated models will have to be considered in practice because the airframe is elastic and will bend. Notch filters on the command signal from the pilot are also used so that the control actions will not excite the bending modes of the airplane.

![Flight envelope of the F4-E. Four different flight conditions are indicated. (From Ackermann (1983), courtesy of Springer-Verlag.)](image)

**Figure 1.13** Flight envelope of the F4-E. Four different flight conditions are indicated. (From Ackermann (1983), courtesy of Springer-Verlag.)

<table>
<thead>
<tr>
<th>Mach</th>
<th>FC 1</th>
<th>FC 2</th>
<th>FC 3</th>
<th>FC 4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.5</td>
<td>0.65</td>
<td>0.9</td>
<td>1.5</td>
</tr>
<tr>
<td>Altitude (feet)</td>
<td>5000</td>
<td>5000</td>
<td>35000</td>
<td>35000</td>
</tr>
<tr>
<td>$a_{01}$</td>
<td>-0.000</td>
<td>-0.002</td>
<td>-0.002</td>
<td>-0.002</td>
</tr>
<tr>
<td>$a_{11}$</td>
<td>-17.41</td>
<td>-19.72</td>
<td>-18.11</td>
<td>-19.18</td>
</tr>
<tr>
<td>$a_{11}$</td>
<td>36.15</td>
<td>207.5</td>
<td>84.34</td>
<td>78.9</td>
</tr>
<tr>
<td>$a_{21}$</td>
<td>0.2448</td>
<td>0.2301</td>
<td>0.2201</td>
<td>0.2101</td>
</tr>
<tr>
<td>$a_{02}$</td>
<td>0.8545</td>
<td>1.415</td>
<td>-0.0057</td>
<td>-1.525</td>
</tr>
<tr>
<td>$a_{12}$</td>
<td>-11.39</td>
<td>-32.99</td>
<td>-8.81</td>
<td>-29.38</td>
</tr>
<tr>
<td>$b_0$</td>
<td>97.76</td>
<td>-272.2</td>
<td>-35.08</td>
<td>-175.6</td>
</tr>
<tr>
<td>$b_1$</td>
<td>-3.07</td>
<td>-4.96</td>
<td>-1.87</td>
<td>-9.87</td>
</tr>
<tr>
<td>$b_2$</td>
<td>1.27</td>
<td>1.76</td>
<td>0.56</td>
<td>4.56</td>
</tr>
</tbody>
</table>

Table 1.1 Parameters of the airplane state model of Eq. (1.4) for different flight conditions (FC)
Some Applications of Adaptive Control

- PSS
- Robots
- Level Control
- Pressure Control
- Flow Control
- Temperature Control
- PH Control

Main Resolutions of Classical Adaptive Control

- Gain Scheduling
- Model Reference Adaptive System (MRAS)
- Self tuning Regulators (STR)
- Self Tuning PID
- Self Oscillating Adaptive Systems (SOAS)
• Gain Scheduling

Parameter Change Using Variables of Process Dynamical Characteristic

![Diagram of control system with components: Controller, Process, Gain Scheduler, Reference Input, Control Signal, Output, Accessory Measurement or Operating Point.]

Figure 1.21 Gain scheduling is an important ingredient in modern flight control systems. (By courtesy of Nowrozi Stock Photo, Inc., Neil Hargrave.)
• **Main Characteristics of a Gain Scheduling Controller**

- Flight Control and Autopilot design
- Open Loop Compensation (Parameter Changes)
- Is Gain Scheduling Controller Adaptive?
- Many Examples of Practical Application In Industry
- Rapid Parameters change (Accessory Measurement)
- Number of Operating Points?

• **PID Auto Tuning**

- Methods based on Transient Response
- Methods based on Relay Feedback
- The Closed Loop Ziegler-Nichols Method
• Model Reference Adaptive Systems

Reference Model: Ideal Process Behavior

- Controller Parameters
- Tuning Mechanism
- Reference Input

2 Loops:
- Inner Loop
- Outer Loop

Main Dilemma: Adaptation Mechanism

- Stable MRAS
- Robust MRAS
• **Self-Tuning Regulators**

- Design Criteria
- Process Parameters
- Controller Parameters
- Reference Input
- Design Block
- System Identification
- Process

(STR)

• **Key Points**

- Direct and Indirect Design Strategy
- 2 Control Loops: Inner Loop and Outer Loop
- Design Block
- Practical Implementations in Industry
- Omitting Design Block
- Certainty Equivalence Principle
• Adaptive Control or Robust Control?

 ✓ Criteria for Adaptive Control Application: Robust Control not Applicable.

 Process Dynamics

 Controller with Time Varying Parameters

 STR, MRAS, PID, And other classical methods

 Robust Control

 Gain Scheduling

 Predictable Change

 Vast changes: Difficulty in Uncertainty Modeling

 Rather accurate uncertainty modeling leading to satisfaction of stability conditions and robust performance

 • Step by Step Adaptive Control

 ✓ A Description of Desired Closed Loop Performance
 ✓ Selection of a Controller With the Adapting Ability and Variable Parameters
 ✓ Choice of Parameter Tuning Mechanisms
 ✓ Implementing Adaptive Control
• **Introduction to System Identification**

  ✓ Online Estimation of a Dynamical System’s Parameters is a key element in Adaptive Control.
  ✓ Issues pertaining to system identification

  - Model Structure Selection: Linear, Nonlinear, Model Order, Model Type
  - Experiment Design: Selection of input for identification
  - Parameter Estimation: Method for parameter estimation is the Least Squares Method
  - Model Validation

• **The System Identification**

  - Off-line
  - On-line
  
  • Off-line identification of dynamical systems

  Least Squares Method

  General Schematic:
• Least Squares Offline Identification

• Gauss:
The sum of squares of the differences between the actually observed (system outputs) and the computed values (model outputs), multiplied by numbers that measure the degree of precision, is Minimum.

• Mathematical Modeling

• Describe the unknown plant model in a form that is suitable for system identification methods.
Real but unknown model

\[ Y(s) = G(s)U(s) \]

\[ A(q)y(t) = B(q)u(t) \]

Inverse Transform

\[ y(t) + a_1 y(t-1) + \cdots + a_n y(t-n) = b_1 u(t+m-n-1) + \cdots + b_m u(t-n) \]

\[ y(t) = -a_1 y(t-1) - \cdots - a_n y(t-n) + b_1 u(t+m-n-1) + \cdots + b_m u(t-n) \]

Regression Model

\[ y(t) = \phi^T (t-1) \theta \]
• Problem: Estimation of $\theta$ so that estimation error $e(t) = y(t) - \hat{y}(t)$ or Residuals are minimum.

• Criteria:

$$V(\theta, t) = \frac{1}{2} \sum_{i=1}^{i} (y(i) - \phi^T(i)\theta)^2$$

• Definitions:

$$Y(t) = [y(1) \cdots y(t)]^T$$
$$E(t) = [e(1) \cdots e(t)]^T$$
$$\Phi^T(t) = [\phi(1) \cdots \phi(t)]$$

• Solution: The Least Squares (LS) Estimation Theorem

Minimizing $\hat{\theta}$ for $V(\theta, t) = \frac{1}{2} \sum_{i=1}^{i} (y(i) - \phi^T(i)\theta)^2$ yields:

$$\Phi^T \Phi \hat{\theta} = \Phi^T Y$$

And if $|\Phi^T \Phi| \neq 0$ this minimum is unique

$$\hat{\theta} = (\Phi^T \Phi)^{-1} \Phi^T Y$$
• A Key Point:

✓ Inversion condition for $\Phi^T \Phi \equiv$ The excitation condition

**EXAMPLE 2.1** Least-squares estimation of static system

Consider the system

$$y(i) = b_0 + b_1 u(i) + b_2 u^2(i) + \varepsilon(i)$$

where $\varepsilon(i)$ is zero mean Gaussian noise with standard deviation 0.1. The system is linear in the parameters and can be written in the form (2.1) with

$$\varphi^T(i) = \begin{pmatrix} 1 & u(i) & u^2(i) \end{pmatrix}$$

$$\vartheta = \begin{pmatrix} b_0 & b_1 & b_2 \end{pmatrix}$$

The output is measured for the seven different inputs shown by the dots in Fig. 2.1. In practice the model structure is usually unknown, and the user must decide on an appropriate model. We illustrate this by estimating parameters of the following models:

Model 1: $y(i) = b_0$
Model 2: $y(i) = b_0 + b_1 u$
Model 3: $y(i) = b_0 + b_1 u + b_2 u^2$
Model 4: $y(i) = b_0 + b_1 u + b_2 u^2 + b_3 u^3$

The different models give a polynomial dependence of different orders between $y$ and $u$. 
Table 2.1 shows the least-squares estimates of the different models together with the resulting loss function. Figure 2.1 also shows the estimated relation between $u$ and $y$ for the different models. From the table it is seen that about the same losses are obtained for Models 3 and 4. The fit to the data points is almost the same for these two models, as is seen in Fig. 2.1.

The example shows that it is important to choose the correct model structure to get a good model. With few parameters it is not possible to get a good fit to the data. If too many parameters are used, the fit to the measured data will be very good but the fit to another data set may be very poor. This latter situation is called overfitting.

Table 2.1 Least-squares estimates and loss functions for the system in Example 2.1 using different model structures.

<table>
<thead>
<tr>
<th>Model</th>
<th>$\hat{b}_0$</th>
<th>$\hat{b}_1$</th>
<th>$\hat{b}_2$</th>
<th>$\hat{b}_3$</th>
<th>$\nu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.85</td>
<td></td>
<td></td>
<td></td>
<td>34.46</td>
</tr>
<tr>
<td>2</td>
<td>0.57</td>
<td>1.09</td>
<td></td>
<td></td>
<td>1.01</td>
</tr>
<tr>
<td>3</td>
<td>1.11</td>
<td>0.45</td>
<td>0.14</td>
<td></td>
<td>0.031</td>
</tr>
<tr>
<td>4</td>
<td>1.13</td>
<td>0.27</td>
<td>0.14</td>
<td>-0.003</td>
<td>0.027</td>
</tr>
</tbody>
</table>

**Figure 2.1** The data represent the measured data points. Resulting models, indicated by the solid lines, based on the least-squares estimates are also given for (a) Model 1, (b) Model 2, (c) Model 3, (d) Model 4.
Online Identification of Dynamical Systems

- **Objective:** To retrieve the dynamical system model at each time sample for utilization in control

- **General Schematics:**

  ![System Identification Diagram]

  - System Parameters
  - System Identification
  - Unknown dynamical system

- **Strategy:** Recursive parameter estimation, that is using data up to time t-1 to calculate the estimation at time t.

  **Recursive Least Squares (RLS)**

  **Recursive Calculations:**
  Requisites,
  \[
  \hat{\theta}(t-1) = \text{LS Estimation up to } t - 1
  \]
  \[
  |\Phi^T \Phi| \neq 0
  \]
  \[
  P(t) \Delta \left[\Phi^T \Phi\right]^{-1}
  \]

  ![Covariance Matrix Diagram]
• RLS Algorithm

\[ \Phi^T \Phi \neq 0 \quad \forall \theta \neq 0, \quad \hat{\theta}(t_0), \text{ and } P(t_0) \text{ given:} \]

Correcting gain

\[ \hat{\theta}(t) = \hat{\theta}(t-1) + K(t)[y(t) - \phi^T(t)\hat{\theta}(t-1)] \]

Estimation error

\[ K(t) = P(t-1)\phi(t)[I + \phi^T(t)P(t-1)\phi(t)]^{-1} \]

Correcting Factor

\[ P(t) = [I - K(t)\phi^T(t)]P(t-1) \]

• Key Points:

✓ RLS is a Kalman Filter for the following system:

\[ \theta(t+1) = \theta(t) \]
\[ y(t) = \phi(t)\theta(t) + e(t) \]

✓ The initial selection of the Covariance Matrix:

\[ P(0) = \alpha I, \quad \alpha = 10^4 \]
Application of RLS in online identification of dynamical systems

General Schematics:

Experimental Conditions

What characteristics should the input signal possess in order to implement system identification with the least squares method?

- Input signal must be persistently exciting (PE).
- When is a signal PE?
- Order of persistent excitation of a PE signal
- Conditions for PE
- Definitions, Theorems and Examples