PID Controller Tuning: A Short Tutorial

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Spring, 2006
Outline

This tutorial is in PDF format with navigational control. You may press SPACE or →, or click the buttons in the lower right corner to move to the next slide. Clicking on the outlined items will take you directly to that section.

Goals and Objectives
What are we going to learn?

Introduction
What is a PID controller?
Why do we want to learn the PID Controller?

Tuning Rules
How does the PID parameters affect system dynamics?
The Ziegler-Nichols tuning rule
What are we going to learn?

The goal of the tutorial is for you to learn about the PID controller and a few basic tuning rules of it. After taking this lesson, you will be able to

1. relate PID controller parameters to step response characteristics of the controlled system, and
2. apply the famous Ziegler-Nichols tuning method to come up with an initial set of working PID parameters for an unknown system.
What is a PID controller?

A PID controller is a simple three-term controller. The letters P, I and D stand for:

- P - Proportional
- I - Integral
- D - Derivative

The transfer function of the most basic form of PID controller, as we use in ME475, is

\[ C(s) = K_P + \frac{K_I}{s} + K_D s = \frac{K_D s^2 + K_P s + K_I}{s} \]

where \( K_P \) = Proportional gain, \( K_I \) = Integral gain and \( K_D \) = Derivative gain.
In this tutorial, we assume the controller is used in a closed-loop unity feedback system. The variable $e$ denotes the tracking error, which is sent to the PID controller. The control signal $u$ from the controller to the plant is equal to the proportional gain ($K_P$) times the magnitude of the error plus the integral gain ($K_I$) times the integral of the error plus the derivative gain ($K_D$) times the derivative of the error.

$$u = K_P e + K_I \int e \, dt + K_D \frac{de}{dt}$$
Why learn the PID controller?

Because PID Controllers are everywhere! Due to its simplicity and excellent if not optimal performance in many applications, PID controllers are used in more than 95% of closed-loop industrial processes.\(^1\) It can be tuned by operators without extensive background in Controls, unlike many other modern controllers that are much more complex but often provide only marginal improvement. In fact, most PID controllers are tuned on-site. Although we are learning all the theories in ME475 to design the controller, the lengthy calculations for an initial guess of PID parameters can often be circumvented if we know a few useful tuning rules. This is especially useful when the system is unknown.

How do the PID parameters affect system dynamics?

We are most interested in four major characteristics of the closed-loop step response. They are

1. Rise Time: the time it takes for the plant output $y$ to rise beyond 90% of the desired level for the first time.
2. Overshoot: how much the the peak level is higher than the steady state, normalized against the steady state.
3. Settling Time: the time it takes for the system to converge to its steady state.
4. Steady-state Error: the difference between the steady-state output and the desired output.
How do the PID parameters affect system dynamics?

The effects of increasing each of the controller parameters $K_P$, $K_I$ and $K_D$ can be summarized as

<table>
<thead>
<tr>
<th>Response</th>
<th>Rise Time</th>
<th>Overshoot</th>
<th>Settling Time</th>
<th>S-S Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_P$</td>
<td>Decrease</td>
<td>Increase</td>
<td>NT</td>
<td>Decrease</td>
</tr>
<tr>
<td>$K_I$</td>
<td>Decrease</td>
<td>Increase</td>
<td>Increase</td>
<td>Eliminate</td>
</tr>
<tr>
<td>$K_D$</td>
<td>NT</td>
<td>Decrease</td>
<td>Decrease</td>
<td>NT</td>
</tr>
</tbody>
</table>

NT: No definite trend. Minor change.
You may want to take notes of this table. It will be useful in the later part of the lesson.
How do we use the table?

Typical steps for designing a PID controller are

1. Determine what characteristics of the system needs to be improved.
2. Use $K_P$ to decrease the rise time.
3. Use $K_D$ to reduce the overshoot and settling time.
4. Use $K_I$ to eliminate the steady-state error.

This works in many cases, but what would be a good starting point? What if the first parameters we choose are totally crappy? Can we find a good set of initial parameters easily and quickly?
The Ziegler-Nichols tuning rule to the rescue

Ziegler and Nichols conducted numerous experiments and proposed rules for determining values of $K_P$, $K_I$ and $K_D$ based on the transient step response of a plant. They proposed more than one method, but we will limit ourselves to what’s known as the first method of Ziegler-Nichols in this tutorial. It applies to plants with neither integrators nor dominant complex-conjugate poles, whose unit-step response resemble an S-shaped curve with no overshoot. This S-shaped curve is called the reaction curve.
Using the reaction curve method

The S-shaped reaction curve can be characterized by two constants, delay time $L$ and time constant $T$, which are determined by drawing a tangent line at the inflection point of the curve and finding the intersections of the tangent line with the time axis and the steady-state level line.
The Ziegler-Nichols Tuning Rule Table

Using the parameters $L$ and $T$, we can set the values of $K_P$, $K_I$ and $K_D$ according to the formula shown in the table below. You may want to take notes of the table as we will use it later in the quiz.

<table>
<thead>
<tr>
<th>Controller</th>
<th>$K_P$</th>
<th>$K_I$</th>
<th>$K_D$</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>$\frac{T}{L}$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>PI</td>
<td>$0.9\frac{T}{L}$</td>
<td>$0.27\frac{T}{L^2}$</td>
<td>0</td>
</tr>
<tr>
<td>PID</td>
<td>$1.2\frac{T}{L}$</td>
<td>$0.6\frac{T}{L^2}$</td>
<td>$0.6T$</td>
</tr>
</tbody>
</table>

These parameters will typically give you a response with an overshoot about 25% and good settling time. We may then start fine-tuning the controller using the basic rules that relate each parameter to the response characteristics.
Summary

This concludes the instruction part of our tutorial. We learned two things about PID controllers.

1. Relationships between $K_P$, $K_I$ and $K_D$ and important response characteristics, of which these three are most useful:
   - Use $K_P$ to decrease the rise time.
   - Use $K_D$ to reduce the overshoot and settling time.
   - Use $K_I$ to eliminate the steady-state error.

2. The Ziegler-Nichols tuning rule (reaction curve method) for good initial estimate of parameters.

Enough of theories and rules now, let’s move on to some hands-on experience!