Analysis and Control of Nonlinear Process Systems

K.M. Hangos
J. Bokor
G. Szederkényi

Springer
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Professor Michael J. Grimble, Professor of Industrial Systems and Director
Professor Michael A. Johnson, Professor of Control Systems and Deputy Director
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University of Strathclyde, Graham Hills Building, 50 George Street, Glasgow G1 1QE, U.K.

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J. Bokor, PhD, DSci
G. Szederkényi, PhD
Systems and Control Laboratory, Computer and Automation Institute,
Hungarian Academy of Sciences, H-1516 Budapest, PO Box 63, Kende u. 13-17,
Hungary
For God had not given us the spirit of fear; but of power, and of love, and of a sound mind.

II. Timothy 1.6
Foreword

Process systems constitute a key aspect of human activity that is closely linked to the three pillars of sustainable development: Economic competitiveness, Social importance (employment, quality of life) and Environmental impact. The future economic strength of societies will depend on the ability of production industries to produce goods and services by combining competitiveness with quality of life and environmental awareness. In the combined effort to minimize waste through process integration and to optimally operate the constructed processes nonlinear behaviours are being exploited. Thus there will be an increasing need for nonlinear process theory to systematically deal with the relatively complex nonlinear issues that appear with the increasing process systems complexity dictated by our technological capability and the competitive demands.

This book serves as a most promising source that combines process systems engineering with nonlinear systems and control theory. This combination is carried through in the book by providing the reader with references to linear time invariant control theory. The nonlinear passivity theory constitutes a particularly promising contribution that is illustrated on problems of relatively low dimensionality.

The successful establishment of the state-of-art in nonlinear process systems control in a concise textbook represents a laudable contribution to process systems theory for the benefit of future graduate students and researchers and hopefully also for the benefit of human activity.

Lyngby, July 2003

Professor Sten Bay Jørgensen
Director of CAPEC (Computer Aided Process Engineering Center)
Department of Chemical Engineering
Technical University of Denmark
Lyngby, Denmark
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Series Editors’ Foreword

The topics of control engineering and signal processing continue to flourish and develop. In common with general scientific investigation, new ideas, concepts and interpretations emerge quite spontaneously and these are then discussed, used, discarded or subsumed into the prevailing subject paradigm. Sometimes these innovative concepts coalesce into a new sub-discipline within the broad subject tapestry of control and signal processing. This preliminary battle between old and new usually takes place at conferences, through the Internet and in the journals of the discipline. After a little more maturity has been acquired by the new concepts then archival publication as a scientific or engineering monograph may occur.

A new concept in control and signal processing is known to have arrived when sufficient material has evolved for the topic to be taught as a specialised tutorial workshop or as a course to undergraduate, graduate or industrial engineers. Advanced Textbooks in Control and Signal Processing are designed as a vehicle for the systematic presentation of course material for both popular and innovative topics in the discipline. It is hoped that prospective authors will welcome the opportunity to publish a structured and systematic presentation of some of the newer emerging control and signal processing technologies in the textbook series.

As most of the problems from linear control analysis have found solutions, advances in future control performance will come from accommodating the non-linear nature of many processes more directly. This is a challenge facing many areas of control engineering. In the process industries there is a fair amount of non-linear model information and the task is to find ways to exploit this knowledge base. On the other hand the analysis of non-linear systems per se is reasonably well developed but in many cases the move to more routine application of these techniques still remains to be taken. We believe it is only by having the utility and advantages of non-linear control demonstrated in practical applications that the non-linear control paradigm will begin to make a contribution to control engineering.
Process control is one area where there is the possibility of demonstrating a major advance through the use of non-linear control. Tackling this challenge we are pleased to have this textbook by Katalin Hangos, József Bokor, and Gábor Szederkényi on “Analysis and control of non-linear process systems” in the Advanced Textbooks in Control and Signal Processing series. It is a text based on past course experience and care has been taken to enhance the accessibility of the material with nice pedagogical features like special indexes, boxed important definitions and end-of-chapter exercises. The underlying rigour of the non-linear analysis has however been preserved. The book is suitable for graduate and postgraduate courses in process systems engineering and for self-study at that level. It is our hope that this textbook will contribute to the more widespread acceptance of non-linear control in applications.

M.J. Grimble and M.A. Johnson
Industrial Control Centre
Glasgow, Scotland, U.K.
Summer 2003
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