STABLE ADAPTIVE CONTROL AND ESTIMATION FOR NONLINEAR SYSTEMS
Adaptive and Learning Systems for Signal Processing, Communications, and Control

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STABLE ADAPTIVE CONTROL AND ESTIMATION FOR NONLINEAR SYSTEMS
Neural and Fuzzy Approximator Techniques

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To our families
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A key issue in the design of control systems has long been the robustness of the resulting closed-loop system. This has become even more critical as control systems are used in high consequence applications in which certain process variations or failures could result in unacceptable losses. Appropriately, the focus on this issue has driven the design of many robust nonlinear control techniques that compensate for system uncertainties.

At the same time neural networks and fuzzy systems have found their way into control applications and in sub-fields of almost every engineering discipline. Even though their implementations have been rather ad hoc at times, the resulting performance has continued to excite and capture the attention of engineers working on today’s “real-world” systems. These results have largely been due to the ease of implementation often possible when developing control systems that depend upon fuzzy systems or neural networks.

In this book we attempt to merge the benefits from these two approaches to control design (traditional robust design and so called “intelligent control” approaches). The result is a control methodology that may be verified with the mathematical rigor typically found in the nonlinear robust control area while possessing the flexibility and ease of implementation traditionally associated with neural network and fuzzy system approaches. Within this book we show how these methodologies may be applied to state feedback, multi-input multi-output (MIMO) nonlinear systems, output feedback problems, both continuous and discrete-time applications, and even decentralized control. We attempt to demonstrate how one would apply these techniques to real-world systems through both simulations and experimental settings.

This book has been written at a first-year graduate level and assumes some familiarity with basic systems concepts such as state variables and stability. The book is appropriate for use as a text book and homework problems have been included.
Organization of the Book

This book has been broken into four main parts. The first part of the book is dedicated to background material on the stability of systems, optimization, and properties of fuzzy systems and neural networks. In Chapter 1 a brief introduction to the control philosophy used throughout the book is presented. Chapter 2 provides the necessary mathematical background for the book (especially needed to understand the proofs), including stability and convergence concepts and methods, and definitions of the notation we will use. Chapter 3 provides an introduction to the key concepts from neural networks and fuzzy systems that we need. Chapter 4 provides an introduction to the basics of optimization theory and the optimization techniques that we will use to tune neural networks and fuzzy systems to achieve the estimation or control tasks. In Chapter 5 we outline the key properties of neural networks and fuzzy systems that we need when they are used as approximators for unknown nonlinear functions.

The second part of the book deals with the state-feedback control problem. We start by looking at the non-adaptive case in Chapter 6 in which an introduction to feedback linearization and backstepping methods are presented. It is then shown how both a direct (Chapter 7) and indirect (Chapter 8) adaptive approach may be used to improve both system robustness and performance. The application of these techniques is further explained in Chapter 9, which is dedicated to implementation issues.

In the third part of the book we look at the output-feedback problem in which all the plant state information is not available for use in the design of the feedback control signals. In Chapter 10, output-feedback controllers are designed for systems using the concept of uniform complete observability. In particular, it is shown how the separation principle may be used to extend the approaches developed for state-feedback control to the output-feedback case. In Chapter 11 the output-feedback methodology is developed for adaptive controllers applicable to systems with a great degree of uncertainty. These methods are further explained in Chapter 12 where output-feedback controllers are designed for a variety of case studies.

The final part of the book addresses miscellaneous topics such as discrete-time control in Chapter 13 and decentralized control in Chapter 14. Finally, in Chapter 15 the methods studied in this book will be compared to conventional adaptive control and to other "intelligent" adaptive control methods (e.g., methods based on genetic algorithms, expert systems, and planning systems).

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This manuscript was prepared using \LaTeX. The simulations and many of the figures throughout the book were developed using MATLAB.

As mentioned above, the material in this book depends critically on conventional robust adaptive control methods, and in this regard it was especially influenced by the excellent books of P. Ioannou and J. Sun, and S. Sastry and M. Bodson (see Bibliography). As outlined in detail in the "For Further Study" section of the book, the methods of this book are also based on those developed by several colleagues, and we gratefully acknowledge their contributions here. In particular, we would like to mention: J. Farrell, H. Khalil, F. Lewis, M. Polycarpou, and L-X. Wang. Our writing process was enhanced by critical reviews, comments, and support by several persons including: A. Bentley, Y. Diao, V. Gazi, T. Kim, S. Kohler, M. Lau, Y. Liu, and T. Smith. We would like to thank B. Codey, S. Paracka, G. Telecki, and M. Yanuzzi for their help in producing and editing this book. Finally, we would like to thank our families for their support throughout this entire project.

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